Ex Ante Valuation of Systemic Risk: a Financial Equilibrium Networks Approach

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Roadmap

Question

- How does network risk affect *ex ante* risk assessments.

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- Deposit insurance premia.

Motivation

The New York Times

Silicon Valley Bank Fails After Run on Deposits

The Federal Deposit Insurance Corporation took control of the bank's assets on Friday. The failure raised concerns that other banks could face problems, too.

By Emily Flitter and Rob Copeland

Emily Flitter and Rob Copeland cover Wall Street and finance.

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What We Do

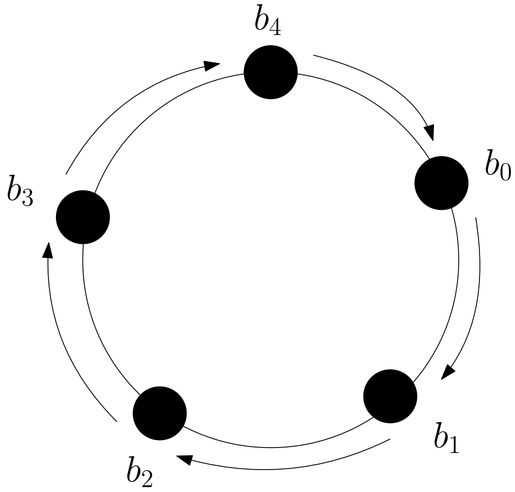
- Develop a tractable financial network model, which facilitates *ex-ante* analysis.

- Compute *ex-ante* risk premia on balance sheet items, inclusive of network risk.

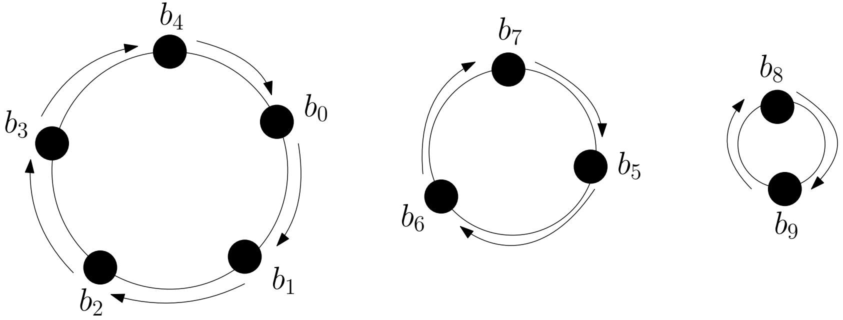
What We Do

- Model has two essential ingredients in measuring risk:
  - Endogenous liquidation values of assets,
  - Priority claims.
- We consider 3 network structures.

What We Do



What We Do



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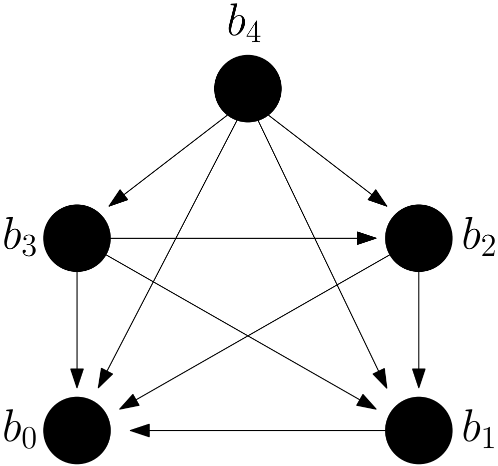

What We Do

- I'll focus on mutiple directed cycles.

Roadmap

Setup

- Refer to agents as banks (use interchangeably with firms).
- Cycles: building on Caballero & Simsek (2013, Journal of Finance).
- One-way directed links: inter-bank loans.
- N symmetric banks in the economy.

Cycle Sizes

- Cycles can have $m \in [2, N]$ banks where

$$\underbrace{N}_{\text{No. banks}} = \sum_{m=2}^{N} m \underbrace{n(m)}_{\text{No. cycles of size } m}$$

$$\underbrace{N^*}_{\text{No. cycles}} = \sum_{m=2}^{N} n(m)$$

Bank Balance Sheet

Assets	**Liabilities**
Revenues (R)	Deposits (F)
Bank Loans (D)	Bank Deposits (D)
Non-liquid Assets (K)	Equity (E)

- Deposits F have priority.
- $E = K + R - F > 0$.

Endogenous Fire Sales

- Liquidation value function

$$L = \ell(\widehat{N})$$

where

  - L is liquidation value
  - $\widehat{N}$ is number of liquidated banks,
  - Function has $\ell'(\widehat{N}) < 0$, $\ell''(\widehat{N}) > 0$ and $\ell(0) = K$.

Assumptions

- ϕ liquidity shocks hit banks where

$$\underbrace{\phi}_{\text{No. shocks}} < \underbrace{N^*}_{\text{No. cycles}}$$

and $R = 0$ for banks hit by shocks.

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- Assumption 1: no aggregate uncertainty (ϕ known).

- Assumption 2: at most one bank in each cycle is hit by a shock. Probability a cycle is hit is independent of size.

Assumptions

- Assumption 3: $\underbrace{R - F}_{\text{Revenues (not shocked) less deposits}} > 0.$

Roadmap

Solvency Condition

Proposition 1

For cycle size m sufficiently large, the number of *additional* banks that are liquidated, v^* (i.e. additional to the initial bank liquidated), satisfies the solvency condition:

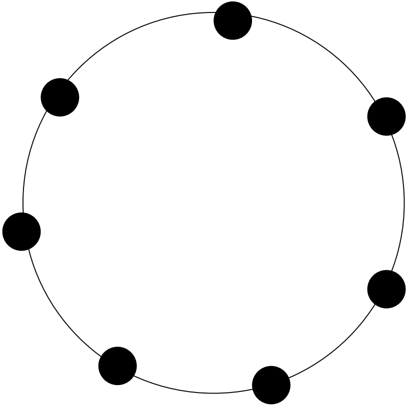
$$v^* R + v^* L - (v^* + 1)\, F < 0 \leqslant (v^* + 1) R + (v^* + 1)\, L - (v^* + 2)\, F.$$

Solvency Condition

- Example where $v^* = 1$.

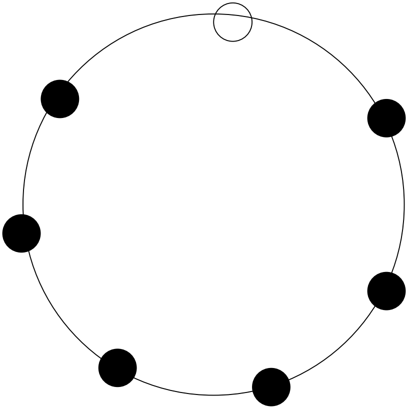
$\Rightarrow$ Solvency condition: $R + L - 2F < 0 \leq 2R + 2L - 3F$.

Solvency Condition



Solvency Condition

$R = 0$
Assets $= D + L$
Liabilities $= D + F$



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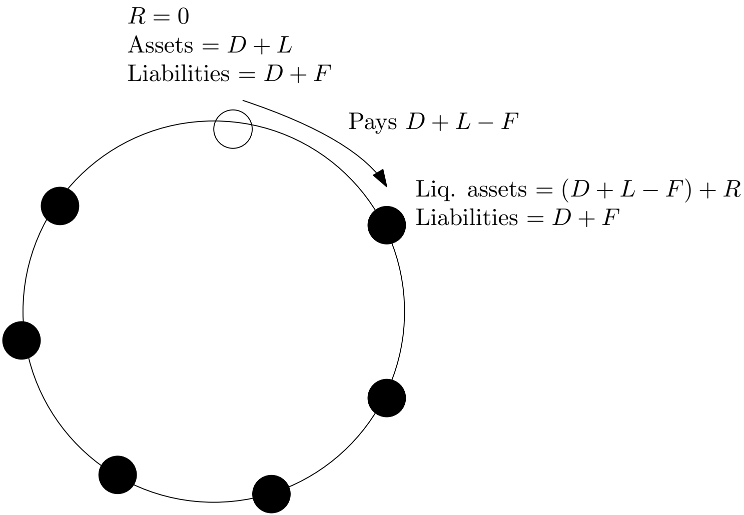

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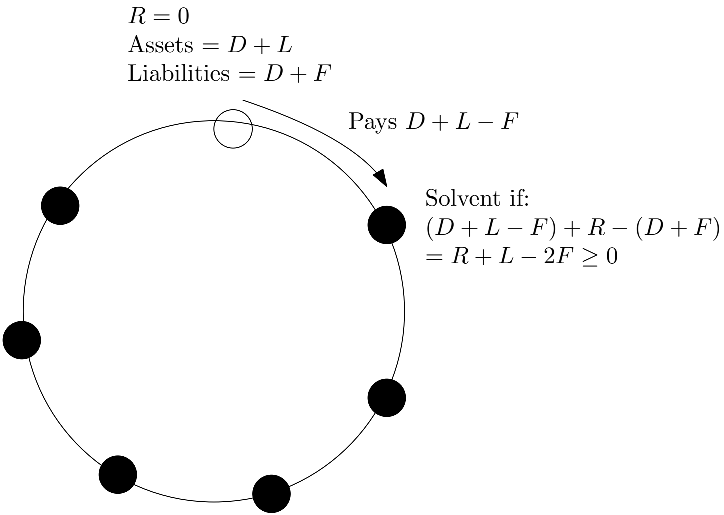

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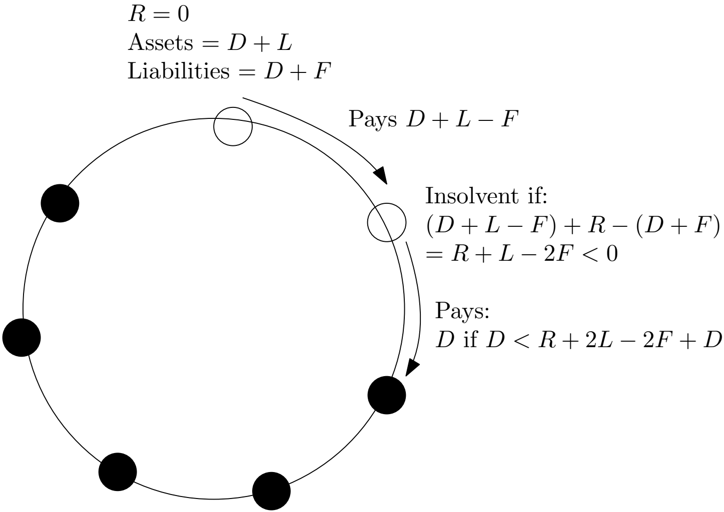

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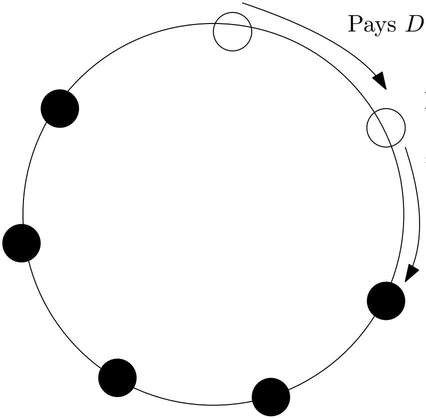

Pays $D + L - F$

Insolvent if:
$(D + L - F) + R - (D + F)$
$= R + L - 2F < 0$

Pays:
D if $D < R + 2L - 2F + D$
$\Rightarrow 0 < R + 2L - 2F$
$\Rightarrow 0 < 2R + 2L - 3F$ by assn. 3

Therefore:
$v^* = 1$ and
$R + L - 2F < 0 < 2R + 2L - 3F$

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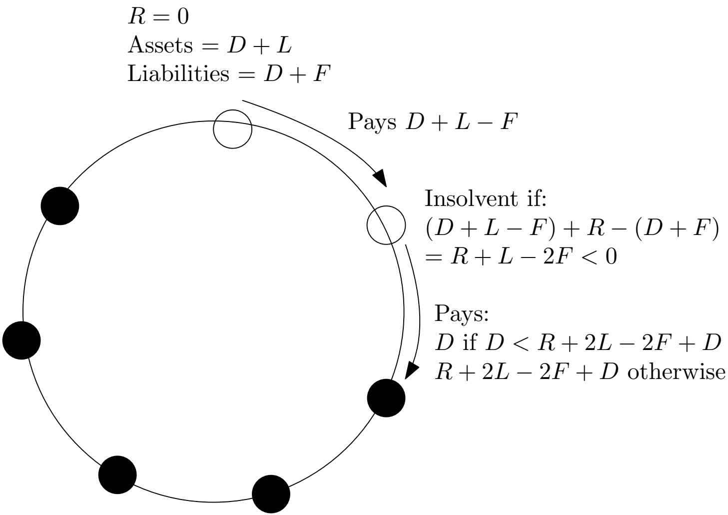

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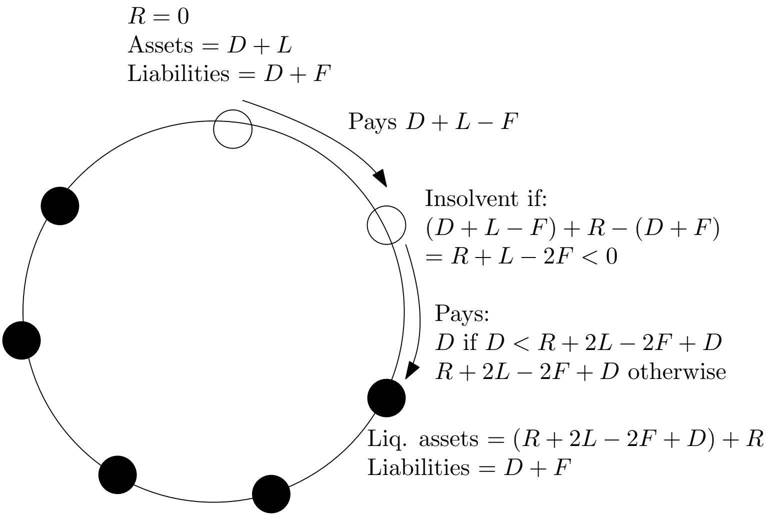

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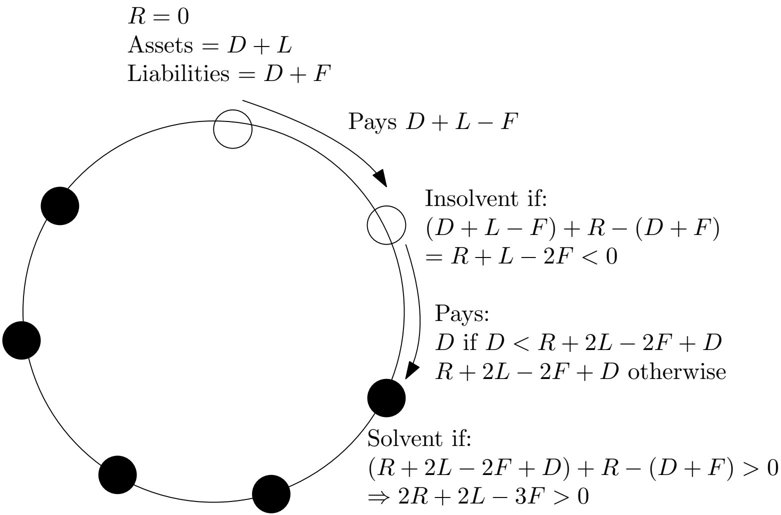

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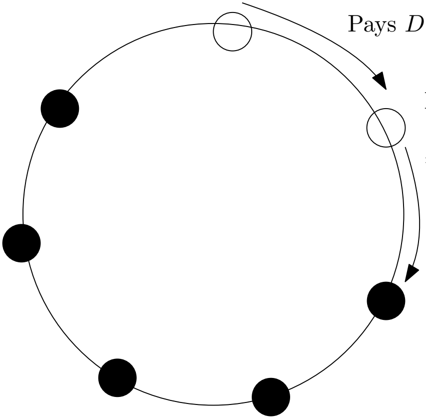


Pays $D + L - F$

Insolvent if:
$(D + L - F) + R - (D + F)$
$= R + L - 2F < 0$

Pays:
D if $D < R + 2L - 2F + D$
$R + 2L - 2F + D$ otherwise

Therefore:
$v^* = 1$ and
$R + L - 2F < 0 < 2R + 2L - 3F$

Losses

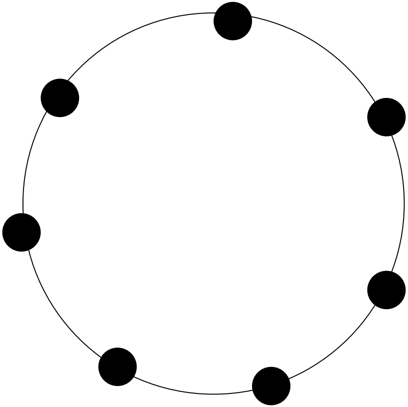
Proposition 2

Suppose that a bank belonging to a cycle of size m is hit by a shock. Then the total number of banks in that cycle that will be liquidated, $\widehat{v}$, is given by

$$\widehat{v} = \begin{cases} v^* + 1 & \text{if } m > v^* \\ m & \text{if } m \leq v^* \end{cases}$$

Losses

$v^* = 3$



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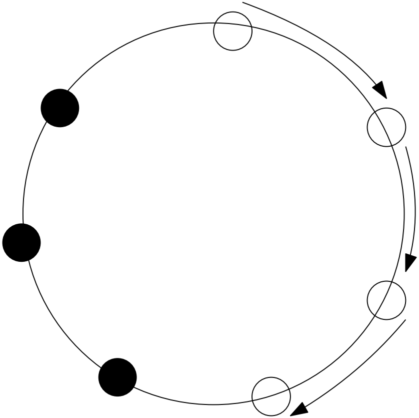

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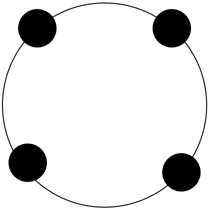

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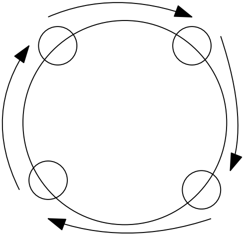

Losses

- Notice though there are distributional issues when $m \leq v^*$.
- Depositors of the bank hit by shock will not be fully compensated.
- Cascade completes revolution around the circle.

Roadmap

Probability of Survival

- Probability of survival $\Rightarrow$ Implies risk premium on each item.

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- Probability of survival $\Rightarrow$ Implies risk premium on each item.
- Denote survival probability π.
- Denote probability of receiving revenues as $\pi_R = 1 - \phi/N$.
- Approximate solution where last surviving bank is repaid in full.

Probability of Survival

- For cases where v^* does not vary with the shock distribution

$$\pi = \underbrace{\left(1 - \frac{\phi}{N^*}\right)}_{\text{Cycle not hit}} + \underbrace{\frac{\phi}{N^*}}_{\text{Cycle hit}} \left(\sum_{m=v^*+2}^{N} \left(\underbrace{\frac{mn(m)}{N}}_{\text{In cycle size } m} \quad \underbrace{\frac{m - [v^* + 1]}{m}}_{\text{Caught in cascade}} \right) \right)$$

for $v^* \in \{0, 1, 2, 3\}$

Premia

- Risk on revenues (R)

$$\rho^R = \underbrace{\frac{1}{\pi_R}}_{\text{Not hit by shock}}$$

- Risk on equity (E)

$$\rho^E = \underbrace{\frac{1}{\pi}}_{\text{Survives}}$$

Premia

- Risk on interbank loans (D)

$$\rho^D = \frac{1}{1 - \underbrace{(\pi_R - \pi)}_{\text{Fails from network effects}}}$$

- Risk on deposits

$$\rho_F \leq \frac{1}{\pi_F}$$

$$\pi_F = \left(1 - \frac{\phi}{N^*}\right) + \frac{\phi}{N^*}\left\{\underbrace{\sum_{m=2}^{v^*+1} \frac{mn(m)}{N}\frac{m-1}{m}}_{\text{Small cycle, not hit}} + \underbrace{\sum_{m=v^*+2}^{N} \frac{mn(m)}{N}}_{\text{Big cycle}}\right\}.$$

Comparative Statics

- Closed-form comparative statics.

- E.g. compare two different networks: G and G'.

- Both have the same total number of firms and shocks.

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- Closed-form comparative statics.
- E.g. compare two different networks: G and G'.
- Both have the same total number of firms and shocks.
- If cascade length constant and same across G and G'

$$\pi(G') - \pi(G) = \frac{\phi}{N} \sum_{m=2}^{v^*+1} (m - [v^* + 1]) \underbrace{\left(\frac{n(m)}{N^*} - \frac{n'(m)}{N^{*\prime}} \right)}_{\text{Fraction of cycles size } m}$$

Numerical example

Roadmap

Summary

- Developed a tractable model facilitating ex-ante risk assessments.
- Risk premia risk change non-linearly with network risk.
- Model sufficiently rich to offer qualitative insights for more general networks.

Numerical Example

- How does changing the configuration of the network affect the distribution of defaults?
- How does ex-ante risk vary with network configuration?

Numerical Example

- Consider the following economy:
  - $N = 100$ banks,
  - $\ell(\widehat{N}) = K\left(1 - \sqrt{\widehat{N}/N}\right)$ liquidation function,
  - $\phi = 3$ shocks,

Numerical Example

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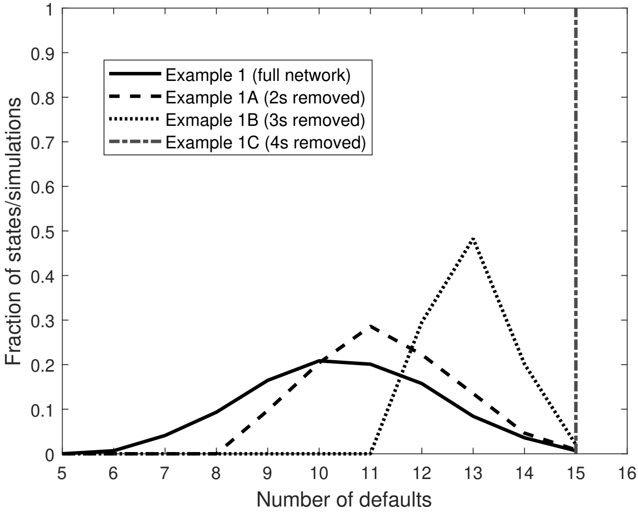
Assets		**Liabilities**	
Revenues:	$R = 1.00$	Deposits:	$F = 0.95$
Bank Loans:	$D = 0.80$	Bank Deposits:	$D = 0.80$
Non-liquid Assets:	$K = 0.25$	Equity:	$E = 0.30$

- Example has $v^* = 4$.

Numerical Example

cycle Size (m)	Number of cycles ($n(m)$) Ex 1	Ex 1A	Ex 1B	Ex 1C
2	6	0	0	0
3	8	12	0	0
4	7	7	16	0
5	3	3	3	3
6	1	1	1	1
7	1	1	1	1
8	1	1	1	9

Numerical Example



Numerical Example

	Ex 1	Ex 1A	Ex 1B	Ex 1C
$\pi_{v^*=0}$	0.970	0.970	0.970	0.970
$\pi_{v^*=4}$	0.896	0.887	0.872	0.850
ρ_R	1.031	1.031	1.031	1.031
ρ_D	1.080	1.090	1.109	1.136
ρ_E	1.117	1.127	1.147	1.176

Back