Lecture 0
Prologue on Numerical Solutions, Coding and Matlab

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Roadmap

Numerical vs Analytical Solutions

- This course will all be about model solving using a computer.
- Why do we need to use computers for such activities at all?
- Complicated models can't be solved with a pen and paper.

Numerical vs Analytical Solutions

- Take the social planner's problem for the neoclassical growth model for example.

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{aligned} c_t + i_t &= f(k_t) \\ i_t &= k_{t+1} - (1-\delta) k_t. \end{aligned}$$

Numerical vs Analytical Solutions

- Solution can be characterised by the Euler equation and resource constraint

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} [1 - \delta + f'(k_{t+1})] \tag{1}$$

$$c_t + k_{t+1} = f(k_t) + (1 - \delta) k_t. \tag{2}$$

- Where does one go from here?

Numerical vs Analytical Solutions

- There are really two options.

(1) Impose assumptions on the problem to get an analytical (pen and paper) solution.

(2) Take it to a computer and solve it numerically.

Numerical vs Analytical Solutions

- What's the tradeoff?
- Numerical solutions usually involve "less restrictive" assumptions.
- But your solution only holds for a specific set of parameters.

Analytical Solutions

- In the context of our neoclassical growth model, we can get analytical solutions under the right assumptions.

- Assume the following

$$u(c_t) = \log(c_t)$$
$$f(k_t) = k_t^{\alpha}$$
$$\delta = 1.$$

- We can then use the guess and verify method to find an analytical solution.

Analytical Solutions

- Conjecture that $k_{t+1} = \omega y_t$ for $\omega \in [0, 1]$ where $y_t = k_t^{\alpha}$.
- Says that your investment is some fraction of final output.
- Get then from equation (2) that $c_t = (1 - \omega) y_t$.
- I.e. consumption takes the remainder of final output.

Analytical Solutions

- Recall equation (1), but now under our assumptions

$$1 = \beta \frac{c_t}{c_{t+1}} \alpha k_{t+1}^{\alpha-1}.$$

- Notice that $\alpha k_{t+1}^{\alpha-1} = \alpha y_{t+1}/k_{t+1}$.

Analytical Solutions

- Follows then that

$$1 = \beta \frac{(1-\omega) y_t}{(1-\omega) y_{t+1}} \alpha \frac{y_{t+1}}{k_{t+1}}$$
$$\Rightarrow 1 = \alpha\beta \frac{y_t}{k_{t+1}}$$
$$\Rightarrow k_{t+1} = \alpha\beta y_t = \alpha\beta k_t^{\alpha}$$

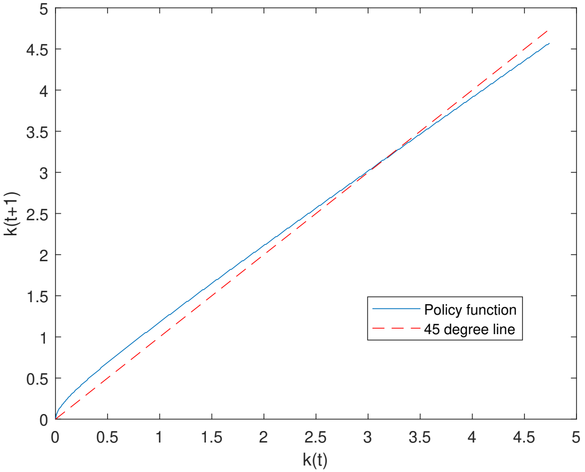
- This is our solution: tell me the current period capital stock k_t and then, using equation (3), I can tell you k_{t+1} through

$$k_{t+1}(k_t) = \alpha\beta k_t^{\alpha}. \tag{3}$$

Numerical

- What if we don't like log utility and full depreciation though?
- Say we want $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ with $f(k_t) = k_t^{\alpha}$.
- Set $\alpha = 0.33$, $\delta = 0.10$, $\sigma = 2.0$ and $\beta = 0.95$.
- Solve for the numerical equivalent of $k_{t+1}(k_t)$ in (3).

Numerical



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
k(t+1)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
k(t)
Policy function
45 degree line

Numerical

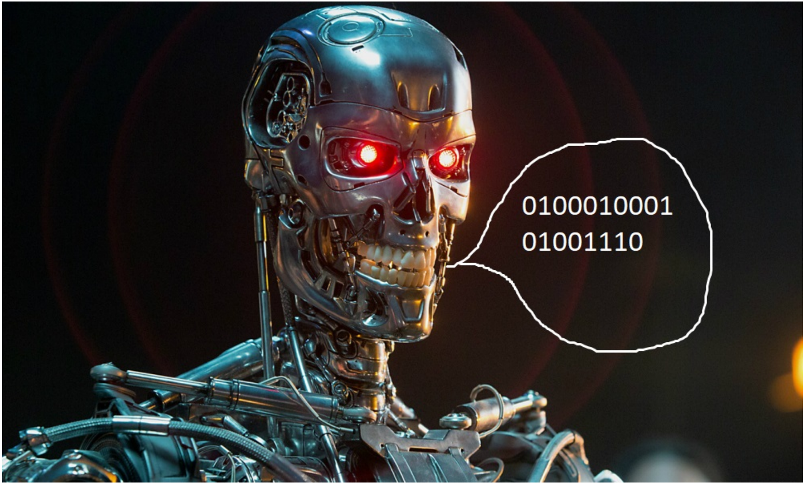
- Finding these numerical solutions is what this course is all about.

Roadmap

Talking to the Machines

- Human language: words, letters and numbers.
- Machine language: binary of 0s and 1s.
- How do we communicate with the machines? Through coding/programming languages.

Talking to the Machines



Languages

- Programming languages take our commands and then translate to binary for the machines to understand.
- Lots of alternatives with different pros and cons.
- Matlab is pretty close to a standard for economics.
- Or at the least, it's a good starting point.

Roadmap

Matlab

- Stands for Matrix Laboratory.
- Basically an interface built over the top of C/C++.
- Matlab always works fastest when you use lots of matrices in your code.

Very Basics

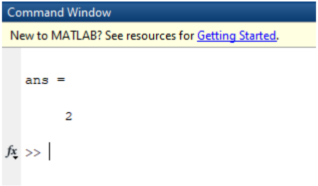
- Matlab script (code) files have .m extensions. E.g. PS1.m.
- We'll proceed in the lecture by example.

Very Basics

- Clear the memory and workspace, then crunch the sum of 1+1

```
clear;clc;
1+1;
1+1
```

gives the output



- The ; suppresses output: i.e. the 1+1 was only printed once when the ; didn't follow.

Arrays

- Best practice is always to declare the size of arrays (vectors) before filling them.
- Declare an array (call it A) of size 3×1 and then fill it with the numbers 1, 2 and 3.

```
A = zeros(3,1);
A(1,1) = 1;
A(2,1) = 2;
A(3,1) = 3;
A
```

gives the output

```
A =

     1
     2
     3
```

Arrays

- Declare an array (call it B) of size 1×3 and then fill it with the numbers 1, 2 and 3.

```
B = zeros(3,1);
B(1,1) = 1;
B(1,2) = 2;
B(1,3) = 3;
B
```

  gives the output

```
B =

     1     2     3
```

- The same idea follows for matrices.

Arrays

- There is a difference between matrix operations and element-by-element operations.
- Declare two matrices of size 2×2. Call them C and D. Fill them both with ones.

```
C = zeros(2,2);
D = zeros(2,2);
C(:,:) = 1;
D(:,:) = 1;
```

Arrays

- Then multiply them together (in the matrix sense).

```
C*D
```

gives the output

```
>> C*D

ans =

     2     2
     2     2
```

Arrays

- Now multiply C and D together element-by-element

  ```
  C.*D
  ```

  gives the output

  ```
  >> C.*D

  ans =

       1     1
       1     1
  ```

  which is like crunching $C(1,1) * D(1,1)$, $C(1,2) * D(1,2)$, ... etc and storing the results in a 2×2 matrix.

For Loops

- These are known as “do loops” in other languages.
- Says to perform an operation several times, where each run is indexed by an integer.

For Loops

- Create a 3×1 array of ones called E. Loop through each element of the array and print the output from multiplying each element by its position number.

```
E = zeros(3,1);
E(:,:) = 1;
for i = 1:length(E);
    i*E(i,1)
end;
```

gives output

```
ans =

     1


ans =

     2


ans =

     3
```

If Statements

- The conditional statement.
- If *something is true* then *do this*.

If Statements

- Using the E array you created, perform the same for loop as before. But for the second entry, instead of printing the entry number, print the number 100.

```
E = zeros(3,1);
E(:,:) = 1;
for i = 1:length(E);
    if (i .eq. 2)
        100
    else
        i*E(i,1)
    end
end;
```

If Statements

- Gives output

```
ans =

     1


ans =

   100


ans =

     3
```

While Loops

- Keep repeating some action *until* some condition is satisfied.

While Loops

- Create a variable called k. Set this variable equal to zero. Keep increasing k by an increment of 1 until it reaches a value of 3. Print the output at each increment.

```
k = 0;
while (k <3)
    k = k + 1
end
```

While Loops

- Yields output

```
i =

     0


i =

     1


i =

     2


i =

     3
```

Functions

- A function is a script that you can call from another, which performs specified tasks.
- It takes *input arguments* and then gives you *outputs*, to which they correspond.
- You need to follow a special syntax in writing the function script to get it to work properly.
- For a function with one input and one output, first line should read

```
function output = myfunction(input)
```

  where *output* (*input*) is the name of your output (input) and *myfunction* is the name you give the script.
- The word function must always the first in the script.
- You must also always save the function script as *myfunction*.m

Functions

- Write a function that takes an input then multiplies it by 100. Call this function from the main body of your script using an input of 10.

- The function script (separate from the main script) would say

```
function output = myfunction(input)
output = input*100
```

  where the main script calls it as follows

```
myfunction(10)
```

  gives

```
>> myfunction(10)

output =

        1000


ans =

        1000
```

Roadmap

Takeaways

- That's more-or-less the basics for us to get started with Matlab.
- Be sure to do problem set 0 before you come to the first lecture!