Lecture 10: New Keynesian Model Part III
New Keynesian Phillips Curve

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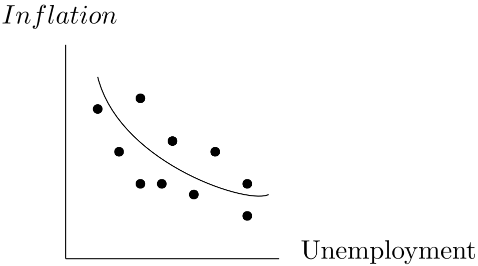
Roadmap

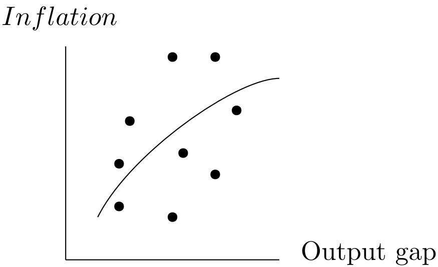
Motivation: Phillips Curve

- Phillips curve: idea that "economic activity" and inflation are positively related.
- Traditionally thought of as unemployment and price inflation having an inverse relationship.



Motivation: Phillips Curve

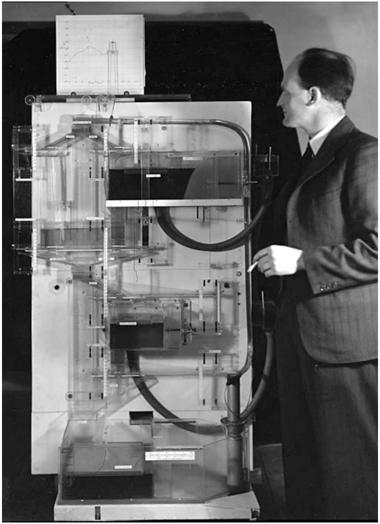
- Can also think of it as positive relationship between the output gap and inflation.
- The economy "heats-up" and prices rise when output is above its natural level.



Motivation: Phillips Curve

- Empirics documented by A.W. Phillips of LSE in the 1950s.

Aside: Phillips and the MONIAC Machine



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Motivation: Phillips Curve and Rational Expectations

- Friedman attacked the Phillips curve due to a lack of proper microfoundations.
- Relationship relies on the idea that you can sustain low unemployment with high inflation eroding real wages.
- But if wage-setters expect high inflation in the future, they'll adjust upwards. Stagflation.

Motivation: Phillips Curve and Rational Expectations

- Where to from here? Researchers tried to build models that would properly account for expectations while preserving the relationship.
- This is what we're after here in this lecture.

Preview of the Punchline

- We already have the ingredients we need to find this object from the last lecture, (the FOC for the optimal pricing problem).
- We just need to linearise it, (as is traditional in this literature).
- In what follows, we'll work with the Calvo pricing model.

Preview of the Punchline

- The object we're working towards is

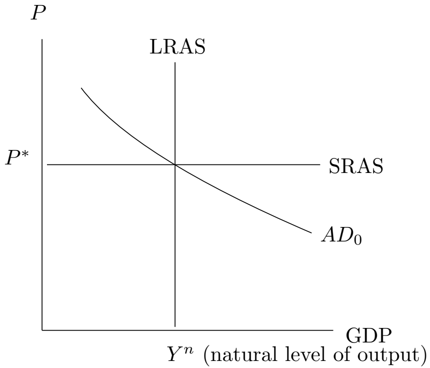
$$\hat{\pi}_t = \kappa \hat{y}_t^g + \beta \mathbb{E}_t[\hat{\pi}_{t+1}]$$

where $\hat{\pi}_t$ is inflation and $\hat{y}_t^g$ is the output gap.

- A rational expectations relationship between inflation and the output gap.

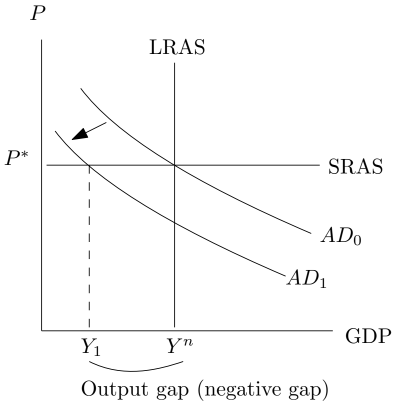
Preview of the Punchline

- Notice that this is like the short run aggregate supply curve: all in temporary deviations.
- Whereas in the long run, prices are perfectly flexible and the natural level of output is all that matters.



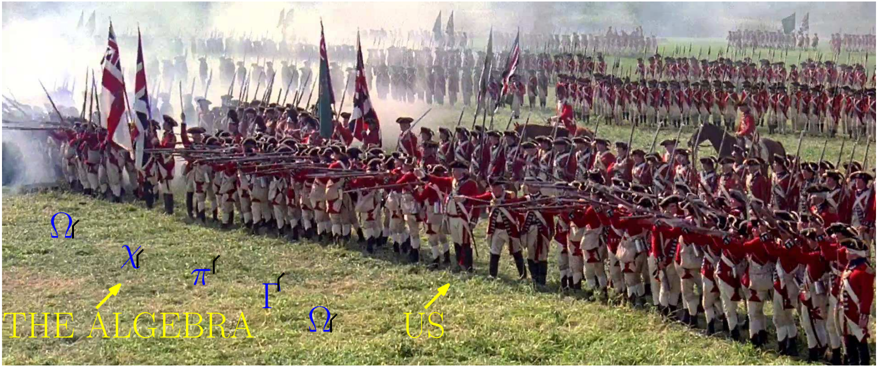
Preview of the Punchline

- Output gap can happen in the short run.
- Gap gets closed with price flexibility in the long run though.



Roadmap

So Begins a War of Algebra...



- It's going to be messy, but the derivation brings up a lot of important concepts.

Pricing FOC

- Recall the pricing FOC (under Calvo) was

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t\to t+k} \left(Y_{t,t+k} \left[(1-\epsilon) + \epsilon \frac{1}{P_t^*} TC'_{t+k}(Y_{t,t+k}) \right] \right) \right\} = 0 \quad (1)$$

where notice that I've dropped the j index and replaced the optimal price with P_t^*, (which is the same across all optimising firms).

Pricing FOC Intuition

- Our objective is to linearise (1).
- Why does this object give us a micro-founded Phillips curve that accounts for expectations?
- Firms set their prices at t expecting that they'll be stuck with it for a while.
- If they anticipate high demand in the future, (positive output gap), then they'll set a higher price than would prevail with perfect price flexibility (in the long run).
- Higher prices generate inflation.
- Positive correlation between expected inflation and the output gap.

Pricing FOC

- Recall that

$$\mathcal{Q}_{t\to t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$$

$$Y_{t,t+k} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

- Substituting these into (1) and re-arranging for P_t^* gives (exercise: hint, you can cancel stuff that doesn't depend on k)

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k})^{-\sigma} P_{t+k}^{\epsilon-1} TC'_{t+k}(Y_{t,t+k}) Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k})^{-\sigma} P_{t+k}^{\epsilon-1} Y_{t+k}}$$

that is — P_t^* satisfies this equation. It's not a solution! Why?

Pricing FOC: Steady State Price Index

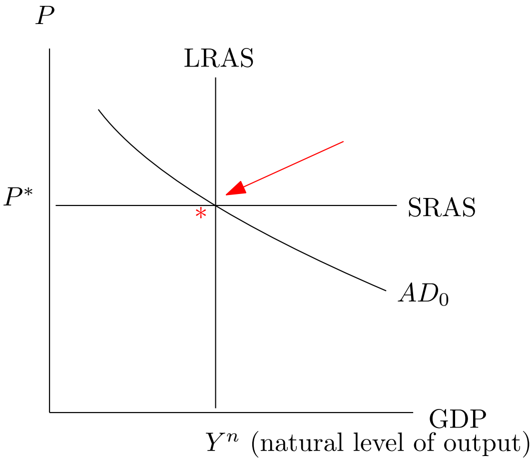
- Recall there will generally be price dispersion with Calvo pricing.
- It's canonical to linearise about a zero inflation steady state.

Pricing FOC: Steady State Price Index

- Zero inflation in the long run.
- What does this mean? See that if $P_t = P_{t-1}$ then

$$
\begin{aligned}
\Rightarrow P_t &= \left[\theta P_t^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
\Rightarrow P_t^{1-\epsilon} &= \theta P_t^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \\
\Rightarrow P_t &= P_t^*
\end{aligned}
$$

Pricing FOC: Steady State Price Index



Pricing FOC: Steady State Price Index

- We'll just sit at point $*$ in the long-run.
- No price changes in steady state.
- Temporary deviations from $*$ induce inflation or disinflation.
- But only in the short-run.
- Right back to natural output (flexible price equilibrium output) in the long-run from these price adjustments.

Pricing FOC: Linearised Price Index

- In linearised form, the pricing law of motion is given by

$$\begin{aligned} \hat{p}_t &= \theta \hat{p}_{t-1} + (1-\theta)\hat{p}^*_t \qquad (2) \\ \Rightarrow \hat{p}_t - \hat{p}_{t-1} &= \theta \hat{p}_{t-1} - \hat{p}_{t-1} + (1-\theta)\hat{p}^*_t \\ \Rightarrow \hat{\pi}_t &= (1-\theta)[\hat{p}^*_t - \hat{p}_{t-1}] \end{aligned}$$

Pricing FOC

- We can then re-write equation (1) as

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}^{-\sigma}) P_{t+k}^{\epsilon-1} Y_{t+k} P_t^* \right\} = \tag{3}$$

$$\frac{\epsilon}{\epsilon - 1} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}^{-\sigma}) P_{t+k}^{\epsilon-1} MC_{t,t+k} Y_{t+k} \right\} \tag{4}$$

where I've denoted $MC_{t,t+k}$ as the marginal cost of a firm at $t+k$ when they set their last optimal price at time t (i.e. $MC_{t,t+k} = TC'_{t+k}(Y_{t,t+k})$).

Pricing FOC: Steady State

- Notice that in steady state, we can write this as

$$\left\{\sum_{k=0}^{\infty}(\theta\beta)^k(\bar{C}^{-\sigma})\bar{P}^{\epsilon-1}\bar{Y}\bar{P}^*\right\} = \frac{\epsilon}{\epsilon-1}\left\{\sum_{k=0}^{\infty}(\theta\beta)^k(\bar{C}^{-\sigma})\bar{P}^{\epsilon-1}\overline{MC}\bar{Y}\right\} \tag{5}$$

  where nothing depends on the k index except for $\sum_{k=0}^{\infty}(\theta\beta)^k = \frac{1}{1-\theta\beta}$.

- Then it follows that (5) simplifies down to

$$\bar{P}^* = \frac{\epsilon}{\epsilon-1}\overline{MC} \tag{6}$$

  what does this say? Look familiar?

Pricing FOC: Log-Linearisation

- Now linearise both sides of (3) to get

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k (\bar{C}^{-\sigma}) \bar{P}^{\epsilon-1} \bar{Y} \bar{P}^* e^{-\sigma \hat{c}_{t+k} + (\epsilon-1)\hat{p}_{t+k} + \hat{y}_{t+k} + \hat{p}_t^*} \right\} \tag{7}$$

$$= \frac{\epsilon}{\epsilon - 1} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k (\bar{C}^{-\sigma}) \bar{P}^{\epsilon-1} \overline{MC} \bar{Y} e^{-\sigma \hat{c}_{t+k} + (\epsilon-1)\hat{p}_{t+k} + \widehat{mc}_{t,t+k} + \hat{y}_{t+k}} \right\}$$

Pricing FOC: Log-Linearisation

- Utilising steady state (6) in equation (7) and expanding the exponentials yields

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \hat{p}_t^* \right\} = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \widehat{mc}_{t,t+k} \right\}$$
$$\Rightarrow \hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \widehat{mc}_{t,t+k} \right\}$$

  what does this say?

- Expressing in terms of real marginal cost, $\widehat{mc}^r_{t,t+k} = \widehat{mc}_{t,t+k} - \hat{p}_{t+k}$ yields

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \left[\widehat{mc}^r_{t,t+k} + \hat{p}_{t+k} \right] \right\} \qquad (8)$$

Roadmap

Where to From Here?

- Equation (8) gives us the optimal reset price as a function of real marginal cost.
- But we want inflation relative to the output gap.
- Find a way to relate $\hat{p}_t^*$ to $\hat{\pi}_t$ and a way to relate the marginal cost to the output gap to finish the job.

Reset Price and Real Marginal Cost (Gap)

- Relate the reset price to real inflation with the time t and time $t+k$ reset prices.
- By studying real marginal cost with a $t+k$ reset, we're getting towards thinking about natural output.

Reset Price and Real Marginal Cost (Gap)

- Recall from lecture 7 that nominal total cost was given as

$$TC(Y) = W\left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}$$

- Follows that the linearised expression for real mc at $t+k$ with time t reset is

$$\begin{aligned}\widehat{mc}^{r}_{t,t+k} &= \widehat{mc}_{t,t+k} - \hat{p}_{t+k} \\ &= \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}\left(\hat{a}_{t+k} - \alpha\hat{y}_{t,t+k}\right)\end{aligned}$$

- Follows that the linearised expression for real mc at $t+k$ with time $t+k$ reset is

$$\begin{aligned}\widehat{mc}^{r}_{t+k} &= \widehat{mc}_{t+k} - \hat{p}_{t+k} \\ &= \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}\left(\hat{a}_{t+k} - \alpha\hat{y}_{t+k}\right)\end{aligned}$$

Reset Price and Real Marginal Cost (Gap)

- The difference can then be written as

$$\widehat{mc}^r_{t,t+k} - \widehat{mc}^r_{t+k} = \frac{\alpha}{1-\alpha}(\hat{y}_{t,t+k} - \hat{y}_{t+k}) \quad (9)$$

- We can then express the demand curve for a given firm as

$$\hat{y}_{t,t+k} = -\epsilon(\hat{p}^*_t - \hat{p}_{t+k}) + \hat{y}_{t+k} \quad (10)$$

- Substitute equation (10) into (9) to obtain

$$\widehat{mc}^r_{t,t+k} = \widehat{mc}^r_{t+k} - \frac{\epsilon\alpha}{1-\alpha}(\hat{p}^*_t - \hat{p}_{t+k}) \quad (11)$$

Reset Price and Real Marginal Cost (Gap)

- Then substitute equation (11) into (8) to get

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\widehat{mc}_{t+k}^r - \frac{\epsilon\alpha}{1-\alpha}(\hat{p}_t^* - \hat{p}_{t+k}) + \hat{p}_{t+k}\right]\right\} \tag{12}$$

$$= (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\widehat{mc}_{t+k}^r + \frac{1-\alpha(1-\epsilon)}{1-\alpha}\hat{p}_{t+k} - \frac{\epsilon\alpha}{1-\alpha}\hat{p}_t^*\right]\right\}$$

$$= (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\widehat{mc}_{t+k}^r + \frac{1-\alpha(1-\epsilon)}{1-\alpha}\hat{p}_{t+k}\right]\right\} - \frac{\epsilon\alpha}{1-\alpha}\hat{p}_t^*$$

$$\Rightarrow \hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\Theta\widehat{mc}_{t+k}^r + \hat{p}_{t+k}\right]\right\}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha(1-\epsilon)}$. So we have an equation relating the optimal reset price to real marginal cost and future prices.

Inflation and Future Inflation

- Next we want to relate inflation to real marginal cost and expected inflation.
- Using equation (12), see that

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\Theta\widehat{mc}_{t+k}^r + \hat{p}_{t+k}\right]\right\} \tag{13}$$

$$= (1-\theta\beta)\left[\Theta\widehat{mc}_t^r + \hat{p}_t\right] + (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=1}^{\infty}(\theta\beta)^k\left[\Theta\widehat{mc}_{t+k}^r + \hat{p}_{t+k}\right]\right\}$$

$$= (1-\theta\beta)\left[\Theta\widehat{mc}_t^r + \hat{p}_t\right] + \theta\beta\mathbb{E}_t[\hat{p}_{t+1}^*]$$

Inflation and Future Inflation

- Now subtract $\hat{p}_{t-1}$ from either side of (13) to yield

$$\begin{aligned}
\hat{p}_t^* - \hat{p}_{t-1} &= (1-\theta\beta)\left[\Theta\widehat{mc}_t^r + \hat{p}_t\right] + \theta\beta\mathbb{E}_t[\hat{p}_{t+1}^*] - \hat{p}_{t-1} \qquad (14)\\
&= (1-\theta\beta)\Theta\widehat{mc}_t^r + (1-\theta\beta)\hat{p}_t + \theta\beta\mathbb{E}_t[\hat{p}_{t+1}^*] - \hat{p}_{t-1}\\
&= (1-\theta\beta)\Theta\widehat{mc}_t^r + \theta\beta\mathbb{E}_t[\hat{p}_{t+1}^* - \hat{p}_t] + \hat{p}_t - \hat{p}_{t-1}\\
&= (1-\theta\beta)\Theta\widehat{mc}_t^r + \theta\beta\mathbb{E}_t[\hat{p}_{t+1}^* - \hat{p}_t] + \hat{\pi}_t\\
\Rightarrow \frac{1}{1-\theta}\hat{\pi}_t &= (1-\theta\beta)\Theta\widehat{mc}_t^r + \theta\beta\frac{1}{1-\theta}\mathbb{E}_t[\hat{\pi}_{t+1}] + \hat{\pi}_t\\
\Rightarrow \hat{\pi}_t &= (1-\theta)(1-\theta\beta)\Theta\widehat{mc}_t^r + \theta\beta\mathbb{E}_t[\hat{\pi}_{t+1}] + (1-\theta)\hat{\pi}_t\\
\Rightarrow \theta\hat{\pi}_t &= (1-\theta)(1-\theta\beta)\Theta\widehat{mc}_t^r + \theta\beta\mathbb{E}_t[\hat{\pi}_{t+1}]\\
\Rightarrow \hat{\pi}_t &= \frac{(1-\theta)(1-\theta\beta)\Theta}{\theta}\widehat{mc}_t^r + \beta\mathbb{E}_t[\hat{\pi}_{t+1}]
\end{aligned}$$

sometimes papers leave it here...

Output Gap

- ...it's more typical to relate real marginal cost to the output gap though. Why would we do this in practice?
- Marginal cost is something that's less observable than output.

Output Gap and Real Marginal Cost

- The measure of the natural level of output comes from the flexible price equilibrium.
- Why? Recall that the model is linearised around the zero inflation steady state.
- This corresponds to the flexible price long-run solution.
- Finding this is just back to the imperfect competition model from a few lectures ago, (but now with dynamics).

Output Gap and Real Marginal Cost: Flexible Prices

- Household labour supply

$$N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{15}$$

- Price setting

$$(1-\alpha) A_t N_t^{-\alpha} = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_t} \tag{16}$$

- Resource constraint

$$Y_t = C_t = A_t N_t^{1-\alpha} \tag{17}$$

Output Gap and Real Marginal Cost: Flexible Prices

- Combining equations (15) and (16) yields

$$N_t^{\varphi} C_t^{\sigma} = \frac{\epsilon - 1}{\epsilon}(1-\alpha) A_t N_t^{-\alpha} \tag{18}$$

- Equations (17) and (18) summarise the flexible price system.

- Exercise: we can obtain the log-linearised natural level of output as

$$\hat{y}_t^n = \frac{1+\varphi}{(1-\alpha)\sigma + \alpha + \varphi}\hat{a}_t \tag{19}$$

...this form makes a lot of sense. Why?

- LRAS depends only on productivity (or productive capacity)!

Real Marginal Cost and Natural Output

- We need to relate y_t^n to real marginal cost somehow.
- Notice that $MC_t^r = \frac{W_t}{P_t}\frac{1}{1-\alpha}\frac{1}{A_t}N_t^{\alpha}$, which means that

$$\begin{aligned}\widehat{mc}_t^r &= \hat{w}_t - \hat{p}_t - \hat{a}_t + \alpha\hat{n}_t \\ &= \varphi\hat{n}_t + \sigma\hat{c}_t - \hat{a}_t + \alpha\hat{n}_t \\ &= \frac{(1-\alpha)\sigma + \alpha + \varphi}{1-\alpha}\hat{y}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t \qquad (20)\end{aligned}$$

where the second line comes from the labour supply condition (15) $(\hat{w}_t - \hat{p}_t = \sigma\hat{c}_t + \varphi\hat{n}_t)$ and the third comes from the production function $(\hat{y}_t = \hat{a}_t + (1-\alpha)\hat{n}_t)$.

Real Marginal Cost and Natural Output

- Notice then that from (19)

$$\begin{aligned} \hat{y}_t^n &= \frac{1+\varphi}{(1-\alpha)\sigma+\alpha+\varphi}\hat{a}_t \\ &= \frac{1+\varphi}{1-\alpha}\frac{1-\alpha}{(1-\alpha)\sigma+\alpha+\varphi}\hat{a}_t \\ \Rightarrow \frac{1+\varphi}{1-\alpha}\hat{a}_t &= \frac{(1-\alpha)\sigma+\alpha+\varphi}{1-\alpha}\hat{y}_t^n \end{aligned} \tag{21}$$

Real Marginal Cost and Natural Output

- Utilising equations (21) and (20) then yields

$$\widehat{mc}_t^r = \frac{(1-\alpha)\sigma + \alpha + \varphi}{1-\alpha}(\hat{y}_t - \hat{y}_t^n) \quad (22)$$

- Substitute (22) into the last line of (14) to get the new Keynesian Phillips curve

$$\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa \hat{y}_t^g$$

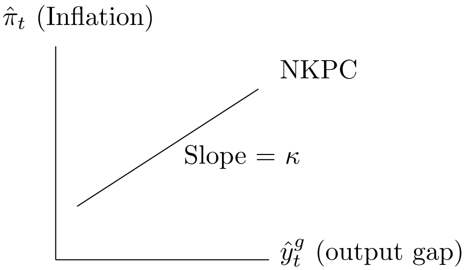
where $\kappa \equiv \frac{(1-\alpha)\sigma+\alpha+\varphi}{1-\alpha}\frac{(1-\theta)(1-\theta\beta)\Theta}{\theta}$ is the slope term and

$$\hat{y}_t^g = \hat{y}_t - \hat{y}_t^n$$

is the output gap.

New Keynesian Phillips Curve

- Notice that $\kappa > 0$.



where the expectation term can be interpreted as a shifter of the curve

Roadmap

Conclusion

- Why did we do all this?
- It's one of the three key equations for the new Keynesian model.
- But this was super elegant. Did you see how neatly the math mapped into the qualitative Keynesian model from L2?
- That's the idea behind this model: mathematical rigour while preserving intuitive economic ideas.
- It's what this discipline should be all about.