# Lecture 16: Financial Intermediation Theory: Bank Runs

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# Roadmap

# Motivation

- Banks typically take liquid deposits (from households) and then hand-out illiquid loans (to businesses or other households).
- Saving allows households to smooth their consumption.
- This can create a maturity mismatch that can be problematic...

# Bank Runs

# Bank Runs

Santander

Nottingham University

- Me to a friend back in Australia: *no banks in Nottingham would take me since I don't have proof of address. But I found a dodgy one on campus that would accept a lower standard of proof called Santander.*
- Friend to me: *nice, you'd better be careful though man, they might go under.*

# Bank Runs

- Diamond-Dybvig (1983), "Bank Runs, Deposit Insurance and Liquidity", *Journal of Political Economy*.

# Intuition

- The paper utilises a simple game-theoretic model where banks balance a tensions between efficient risk sharing and the possibility of runs.
- Depositors may get a bad shock (e.g. lose their job), which would require them to withdraw their funds from the bank.
- There end up being two equilibria: one where there is efficient risk sharing and everything is happy.
- Another where depositors all panic and run to withdraw their deposits.

# Roadmap

# Setup: Depositors

- Three time periods $t \in \{0, 1, 2\}$.
- Unit mass of ex-ante identical depositors and a single bank.
- Each depositor has endowment of 1 to invest at $t = 0$.
- Idiosyncratic shocks drawn by the depositors at $t = 1$.
  - Fraction $s$ are impatient and want to consume at $t = 1$.
  - Fraction $1 - s$ are patient and can consume either at $t = 1$ or $t = 2$.
  - An individual's type is private information but the fraction $s$ is known publicly.
- Assume CRRA preferences $c^{1-\sigma}/(1-\sigma)$ with $\sigma \geqslant 1$.

# Setup: Asset

- The bank takes the depositor's funds and invests them into an illiquid asset.
- Denote the returns from liquidating this asset at time $t \in \{1, 2\}$ by $r_t$.
- If you liquidate that asset at $t = 1$, you get no return, (i.e. $r_1 = 1$).
- If you liquidate at $t = 2$ you get a positive return, (i.e. $r_2 = R > 1$).
- The issue will be that some depositors will want to withdraw their funds at $t = 1$, (impatient types), before the asset has generated a positive return.

# Roadmap

# Optimal Contract

- If a benevolent social planner could offer an optimal insurance contract, what would it look like?
- I.e. how much should impatient and patient depositors get to consume, ($c_1$ and $c_2$ respectively): planner chooses these.
- Subject to only physical constraints: maximise welfare subject to a resource constraint.
- Also need an incentive compatibility constraint since this is now a contracting problem with information asymmetry.

# Optimal Contract

- Solve the problem

$$\max_{c_1, c_2} \; s\left\{\frac{c_1^{1-\sigma}}{1-\sigma}\right\} + (1-s)\left\{\beta\frac{c_2^{1-\sigma}}{1-\sigma}\right\}$$

subject to

$$sc_1 + (1-s)\frac{c_2}{R} \leqslant 1$$
$$\frac{c_1^{1-\sigma}}{1-\sigma} \leqslant \beta\frac{c_2^{1-\sigma}}{1-\sigma}$$

where the first constraint is the resource constraint and the second is the incentive compatibility constraint.

- Where does the resource constraint come from?

# Optimal Contract

- Recall that there was a unit endowment.
- Early withdrawals require $sc_1$.
- What remains will go into the long-term asset with the return. I.e.

$$(1 - s)c_2 \leqslant R[1 - sc_1]$$

where the left-side is total consumption of patient households and the right-side is what goes into the long-term asset, (net of the early withdrawals).

# Constraints

- Recall the two constraints were given by

$$sc_1 + (1-s)\frac{c_2}{R} \leqslant 1$$
$$\frac{c_1^{1-\sigma}}{1-\sigma} \leqslant \beta\frac{c_2^{1-\sigma}}{1-\sigma}$$

- Resource constraint says we divide-up the unit endowment between the early withdrawals and the (discounted) late withdrawals.

- Incentive compatibility constraint says that the patient households are better off waiting until $t = 2$ to consume than to consume at $t = 1$.

# Optimal Contract

- Lagrangian given by

$$\mathcal{L} = s\left\{\frac{c_1^{1-\sigma}}{1-\sigma}\right\} + (1-s)\left\{\beta\frac{c_2^{1-\sigma}}{1-\sigma}\right\} + \lambda_1\left[1 - sc_1 - (1-s)\frac{c_2}{R}\right] + \\ + \lambda_2\left[\beta\frac{c_2^{1-\sigma}}{1-\sigma} - \frac{c_1^{1-\sigma}}{1-\sigma}\right]$$

# Optimal Contract

- FOCs given by

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \Rightarrow sc_1^{-\sigma} - \lambda_1 s - \lambda_2 c_1^{-\sigma} = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 \Rightarrow (1-s)\beta c_2^{-\sigma} - \lambda_1 (1-s)\frac{1}{R} + \lambda_2 \beta c_2^{-\sigma} = 0$$

# Optimal Contract

- No need to show this, (but take my word for it): the incentive compatibility constraint is slack $\Rightarrow \lambda_2 = 0$.
- That is: $\lambda_2 = 0 \Rightarrow \frac{c_1^{1-\sigma}}{1-\sigma} < \beta \frac{c_2^{1-\sigma}}{1-\sigma}$.
- Follow-through with the implications of this to get restrictions or a contradiction. The $c_1$ FOC says

$$\Rightarrow s c_1^{-\sigma} - \lambda_1 s = 0$$
$$\Rightarrow \lambda_1 = c_1^{-\sigma}$$

which can be substituted-into the $c_2$ FOC to get

$$\Rightarrow \beta c_2^{-\sigma} - [c_1^{-\sigma}] \frac{1}{R} = 0$$
$$\Rightarrow c_2 = c_1 \left(\frac{\beta}{R}\right)^{-\frac{1}{\sigma}}$$

# Optimal Contract

- Assume that $\beta = 1$: so now no discounting.
- Also assume that households are quite risk averse $\sigma > 1$.
- Notion of patience and impatience relates purely to this idea of whether they have to consume now or not.

$$c_2 = c_1 R^{\frac{1}{\sigma}}$$

where $c_2 > c_1$ given that $\sigma > 1$ and $R > 1$.

# Optimal Contract

- Back-out $c_1$ and $c_2$ optimal from the resource constraint to get

$$c_1 = \frac{1}{s + (1-s)R^{\frac{1-\sigma}{\sigma}}} \geqslant 1$$
$$c_2 = \frac{R^{\frac{1}{\sigma}}}{s + (1-s)R^{\frac{1-\sigma}{\sigma}}} \leqslant R$$

- This is good: insuring against risk (of needing early withdrawal).

- The autarky allocation is unit consumption for impatient at $t = 1$ and $R$ consumption at $t = 2$ for patient depositors.

- Insurance: consumption smoothing better facilitated!

# Roadmap

# Implementation

- Bank takes deposits (liquid liability) and invests them in asset (illiquid) with payoff $R$ at date $t = 2$.

- Use the following deposit contract:
    - Take deposit of 1 at $t = 0$,
    - Pay $r_1$ to depositors who withdraw early at $t = 1$,
    - Pay $r_2$ to depositors who withdraw late at $t = 2$.

- Check feasibility:
    - Need $sr_1$ funds at $t = 1$.
    - The remaining $1 - sr_1$ is divided up amongst patient depositors $r_2 = \max\left(0, R\frac{1-sr_1}{1-s}\right)$.

# Implementation

- Set the early return $r_1 = c_1$ from the social planner's problem.
- From the resource constraint, you'll get that

$$r_2 = R\frac{1 - r_1}{1 - s}$$

meaning that we can implement the optimal contract using deposits!

# Implementation

- This is only one Nash equilibrium of the deposit game.
- Unfortunately there is also a bank run Nash equilibrium.
- All types might panic at $t = 1$ and withdraw early.

# Bank Runs

- Suppose some fraction $\eta$ withdraw at $t = 1$.
- Return at $t = 2$ then depends on $\eta$.
- $r_2(\eta) = \max\left[0, R\frac{1-\eta r_1}{1-\eta}\right]$.
- Impatient types will always withdraw due to their preferences, so $\eta \geqslant s$.
- Patient types also find it optimal to withdraw when

$$r_2(\eta) < r_1$$
$$\Rightarrow \eta \geqslant \frac{1}{r_1}\frac{R - r_1}{R - 1}$$

where $\eta < 1 \iff r_1 > 1$.

# Bank Runs

- Since $r_1 > 1$, it follows that there are two Nash equilibria!
  - **(i)** Regular times: $\eta = s$ and $r_2(s) = c_2$ in the optimal contract.
  - **(ii)** Bank run: $\eta = 1$ and $r_2(1) = 0$.

# Suspension of Convertibility

- If the bank can commit to stop letting-out withdrawals at $t = 1$, then there is no issue.
- Hard to be credible though...
- Deposit insurance by the government can achieve this!

# Roadmap

# Takeaways

- We can, in principle, implement the optimal risk sharing contract that the social planner chooses using intermediation.
- The maturity mismatch can create problems though.
- We get another Nash equilibrium where a bank run takes place and it goes bust.