Topic 3
Solving Heterogeneous Agent General Equilibrium Models with Idiosyncratic Uncertainty

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Roadmap

1. Introduction
2. Occasionally Binding Constraints
3. RCE in Heterogeneous Agent Models
4. Conclusion

Motivation

- We're all used to seeing the representative agent framework.
- Such a setup need not assume that there is literally only one agent, but rather a degenerate distribution across agents.
- I.e. a representative agent setup abstracts from thinking about cross-sectional dispersion.

Motivation

- But the cross-section can have implications for aggregates!
- Especially in policy settings.
- E.g. HANK (heterogeneous agent new Keynesian) models.
- Kaplan, Moll and Violante (2018, AER), "Monetary Policy According to HANK"...

Motivation

We revisit the transmission mechanism from monetary policy to household consumption in a Heterogeneous Agent New Keynesian (HANK) model. The model yields empirically realistic distributions of wealth and marginal propensities to consume because of two features: uninsurable income shocks and multiple assets with different degrees of liquidity and different returns. In this environment, the indirect effects of an unexpected cut in interest rates, which operate through a general equilibrium increase in labor demand, far outweigh direct effects such as intertemporal substitution. This finding is in stark contrast to small- and medium-scale Representative Agent New Keynesian (RANK) economies, where the substitution channel drives virtually all of the transmission from interest rates to consumption. Failure of Ricardian equivalence implies that, in HANK models, the fiscal reaction to the monetary expansion is a key determinant of the overall size of the macroeconomic response.

Motivation

- Yours truly also wrote a job market paper (a long time ago) looking at the effect of policy changes in the face of firm heterogeneity.
- For the particular policy I studied, the change has close to no effect on the economy in a representative firm framework.
- Does in fact have a quantitatively significant impact with heterogeneous firms and selection effects.
- Changes in the cross-section can aggregate to affect the macroeconomy!

Motivation

- In this course, we'll focus mainly on studying heterogeneity on the household side.
- The concepts extend relatively easily to the firm-side.
- A really good source for firm stuff is Chris Edmond's website.
- Chris is the man.

Motivation

- Household models of heterogeneity are often referred to as incomplete markets models.
- Why? Because one can show a model with heterogeneity and complete markets is isomorphic to a representative agent economy.

Motivation

- Bewley incomplete markets models made quantitative by Hugget and Aiyagari:
  - Huggett, (1993), "The Risk-Free Rate in Heterogeneous Agent Incomplete-Insurance Economies", *Journal of Economic Dynamics and Control*, 17, pp. 953–969.
  - Aiyagari (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving", *Quarterly Journal of Economics*, 109(3), pp. 659–684.

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Borrowing Constraints

- Before we jump-into one of these GE idiosyncratic agent models, let's think a bit more about constraints.
- The households can borrow and save through one period discount bonds.
- Savings denoted by $a_t \geqslant 0$.
- Borrowing denoted by $a_t < 0$.
- Discount bond price at time t given by $q_t < 1$.

Borrowing Constraints

- There must be some limit to borrowing, otherwise these self-obsessed agents will start running ponzi schemes!
- Easiest is to assume some exogenous debt limit

$$a_t \geqslant \underline{a}$$

for some $\underline{a} \leqslant 0$.

Household Problem

- Household solves

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$c_t + q_t a_{t+1} \leqslant a_t + y$$
$$a_{t+1} \geqslant \underline{a}$$

where y is some fixed endowment of income.

Borrowing Constraints

- Bellman equation with the constraint

$$V(a) = U(a + y - c - qa') + \beta V(a') + \mu(a' - \underline{a})$$

where $\mu \geqslant 0$ is the Lagrange multiplier on the borrowing constraint.

Borrowing Constraints

- FOC

$$\frac{\partial V(a)}{\partial a'} = U'(c)(-q) + \beta V'(a') + \mu = 0$$

- Envelope condition

$$V'(a) = U'(c)$$
$$\Rightarrow V'(a') = U'(c')$$

Borrowing Constraints

- Euler equation

$$qU'(c) = \beta U'(c') + \mu$$
$$\Rightarrow U'(c) = q^{-1}\beta U'(c') + q^{-1}\mu$$

or can alternatively think of this as an Euler inequality

$$U'(c) \geqslant q^{-1}\beta U'(c')$$

- We call these constraints occasionally binding, since $\mu = 0$ sometimes, meaning we get our traditional Euler equation back.

Borrowing Constraints: Implementation

- How do we account for the constraint in our computations?
- These constraints look intimidating. They're no big deal though...
- ...provided that you're coding the model up yourself.

Borrowing Constraints: Implementation

- When we code-up the model ourselves, accounting for an occasionally binding constraint just involves an extra 3–4 lines of code.
- Just manually adjust the value function such that the constraint is accounted for.

Borrowing Constraints: Implementation

- In general, assume a household's control x' must be above some variable $\underline{x}(x)$.
- I.e. a function of your current state x.

Value Function Iteration and Constraints

- Augment the VFI algorithm. Say we use gridsearch.
- If our value function is of the form

$$V(x,a) = \max_{x' \geqslant \underline{x}(x)} u(x,x',a) + \beta \mathbb{E}_{a'}[V(x',a')]$$

- Say we have an initial guess of $V_0(x,a)$.
- Find all the values associated with each candidate $x' \in \mathscr{X}$ where

$$\tilde{V}_1(x,x',a) = u(x,x',a) + \beta \mathbb{E}_{a'}[V_0(x',a')] \text{ if } x' \geqslant \underline{x}(x)$$
$$= -\infty \text{ if } x' < \underline{x}(x)$$

then the new value function is

$$V_1(x,a) = \max_{x'} [\tilde{V}_1(x,x',a)].$$

Value Function Iteration and Constraints

- Everything is the same as before, except we need to adjust the value function to prohibit choices that violate the constraint.
- Just use an if statement.
- If *constraint violated* then *new value function is really really negative (e.g. -100000)*.
- Agents will avoid these control choices since it leads to really negative utility. Easy!
- Obviously though, if you have an exogenous lower bound (say $\underline{a}$ for assets), then you can just choose the lower bound on your grid to be that (as we will in the problem set).

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Setup

- We have all the ingredients to solve for a stationary equilibrium of one of these models.
- Huggett's (1993) model studies an endowment economy, while Aiyagari's (1994) studies a production economy.
- Today we'll talk about an endowment economy: I will mention a couple of features of Aiyagari (1994) though.
- We'll go into the production economy in a bit more detail in the next lecture.

Setup

- Assume a unit measure of agents.
- Endowment economy with the usual preferences over consumption

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

Setup

- Idiosyncratic uncertainty with regard to employment status/earnings.
- Two states: employed (e) and unemployed (u).
- Denote a households' state for a given period by $s_t \in \{e, u\}$.
- Denote their earnings by $y_t(s_t)$.
- If $s_t = e$ then $y_t(e) = 1$.
- If $s_t = u$ then $y_t(u) = b < 1$.
- Assume a Markov transition process across the states

$$\pi(s'|s) = \text{Prob}(s_{t+1} = s'|s_t = s)$$

Setup

- Again assume that households can save through discount bonds subject to limit on borrowing.
- No aggregate uncertainty: means that q will be constant in the RCE.
- We'll solve for q endogenously such that there is equilibrium in the bond market.

Household Problem

- Household solves

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$c_t + q_t a_{t+1} \leqslant a_t + y_t$$
$$a_{t+1} \geqslant \underline{a}$$

$\forall t, s_t$. Notice that income changes over time now.

Natural Borrowing Limit

- While Huggett (1993) takes $\underline{a}$ as given, Aiyagari (1994) allows households to borrow up to their natural borrowing limit.
- This is defined as the most that can be borrowed and repaid with probability one across all possible histories.
- See my slides from 2020 for more details.
- We'll just assume the limit is exogenous.

Household's Recursive Problem

- We can write the Bellman equation as

$$v(s,a) = \max_{a'} U(y(s) + a - qa') + \beta \mathbb{E}_{s'}[v(s',a')]$$

where $a' \in \left[\underline{a}, \frac{y(s)+a}{q}\right)$.

- Note that the upper-limit is to ensure the period utility function is properly defined.

- The solution to the problem will be a policy function for the asset control $a' = g(s,a)$.

Cross-Sectional Distribution

- Everything's been pretty standard so far.
- I.e. recursive household problem: thus far all we've done is add-in an occasionally binding borrowing constraint.
- But notice that households in the population will generally differ in their asset holdings, since their states will generally differ.
- Therefore we need to track what the cross-sectional dispersion is over the state space.

Cross-Sectional Distribution

- Denote the measure over assets and employment status at time t by $\mu_t(\mathscr{S}, \mathscr{A})$ where $\mathscr{S}$ and $\mathscr{A}$ are the spaces for employment and assets respectively.
- Notice that I used the word measure.
- In a general RCE, this measure need not integrate to unity.
- E.g. if there is entry and exit of agents into the model: its integral may sum to a number greater than one.
- In this instance though, it'll integrate to one since we assumed a unit mass of agents: we can interpret μ_t as a probability distribution.

Cross-Sectional Distribution

- Exact law of motion.
- Takes as inputs $\mu(s,a)$ current distribution, $g(s,a)$ policy function and transition probability $\pi(s'|s)$ and gives output

$$\mu'(s',a') = \int_{s,a} \mathbb{1}_{a'=g(s,a)} \pi(s'|s)\mu(s,a) ds\, da \tag{1}$$

- What's the measure of agents at (s',a')?
    - It's the measure from last period who transition there.
    - Effectively an endogenous Markov transition probability.
- Note that the integral is needed since many (s,a) combinations can potentially transition to (s',a').

Cross-Sectional Distribution

- We can iterate on (1) to find the steady state measure.
- Give an arbitrary initial distribution (e.g. uniform).
- Iterate until $\mu_t(s, a) \rightarrow \mu(s, a)$.
- I.e. until the distribution isn't changing.

Cross-Sectional Distribution

- Let's think about solving for the stationary distribution.
- Assume that $|\mathcal{S}| = n_S$ and $|\mathscr{A}| = n_A$ (discretised).

Cross-Sectional Distribution: Recipe for Steady State

(1) Solve VFI to get $g(s, a)$ (*ap_policy_ind*).

(2) Give initial distribution $\mu_0(s, a)$ (*mu*). E.g.

$$\mu_0(s, a) = \frac{1}{n_A n_S}, \forall s \in \{1, ..., n_S\}, a \in \{1, ..., n_A\}$$

(3) Crunch the sum (discretised analogue of the integral) in (1)

```
mup = zeros(n_A,n_S);
for a_ind = 1:n_A
 for s_ind = 1:n_S
  for sp_ind = 1:n_S
mup(ap_policy_ind(a_ind, s_ind), sp_ind) = ...
 mup(ap_policy_ind(a_ind, s_ind), sp_ind) + Pi(s_ind, sp_ind)*mu(a_ind, s_ind)
  end
 end
end
```

gives update $\mu_1(s, a)$ (*mup*). Note (*Pi*) is the transition matrix.

(4) Check the distance $||\mu_1(s, a) - \mu_0(s, a)||_\infty$.

(5) Update $\mu_0(s, a) = \mu_1(s, a)$ and repeat until convergence.

Cross-Sectional Distribution: Example

- Let's forget about assets A for a second and just follow some exogenous process s.
- If s is over a continuum:

$$\mu_{t+1}(s') = \int_s \pi(s'|s)\mu_t(s)ds$$

- Let's simplify and say $s \in \{s_1, s_2\}$.
- Law of motion would be

$$\begin{aligned}\mu_{t+1}(s') &= \mu_t(s_1)\pi(s'|s_1) + \mu_t(s_2)\pi(s'|s_2)\\ &= \sum_{s\in\{s_1,s_2\}} \pi(s'|s)\mu_t(s)\end{aligned}$$

Cross-Sectional Distribution: Example

- Let's parameterise with the following

$$\pi = \begin{bmatrix} 0.50 & 0.50 \\ 0.20 & 0.80 \end{bmatrix}$$

$$\mu_t = \begin{bmatrix} 0.10 \\ 0.90 \end{bmatrix}$$

- Then

$$\begin{aligned} \mu_{t+1}(1) &= \mu_t(1)\pi(1,1) + \mu_t(2)\pi(2,1) \\ &= (0.1) * (0.5) + (0.9) * (0.2) \\ &= 0.05 + 0.18 = 0.23 \end{aligned}$$

$$\begin{aligned} \mu_{t+1}(2) &= \mu_t(1)\pi(1,2) + \mu_t(2)\pi(2,2) \\ &= (0.1) * (0.5) + (0.9) * (0.8) \\ &= 0.05 + 0.72 = 0.77 \end{aligned}$$

Cross-Sectional Distribution: Example

- Another way to code this is with loops like in the above.

```
mup = zeros(n_S,1);
for s_ind = 1:n_S
 for sp_ind = 1:n_S
  mup(sp_ind) = mup(sp_ind) + mu(s_ind)*Pi(s_ind, sp_ind)
 end
end
```

- This crunches the expressions above, one piece at a time.
- In our example with $s \in \{s_1, s_2\}$, we have $2x2 = 4$ loop combinations.
- Let's go through them one at a time.

Cross-Sectional Distribution: Example

- For $s\_ind = 1$ and $sp\_ind = 1$

$$\mu_{t+1} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.00 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.00 \end{bmatrix}$$

- For $s\_ind = 1$ and $sp\_ind = 2$

$$\mu_{t+1} = \begin{bmatrix} 0.05 \\ 0.00 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix}$$

- For $s\_ind = 2$ and $sp\_ind = 1$

$$\mu_{t+1} = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.18 \\ 0.00 \end{bmatrix} = \begin{bmatrix} 0.23 \\ 0.05 \end{bmatrix}$$

- For $s\_ind = 2$ and $sp\_ind = 2$

$$\mu_{t+1} = \begin{bmatrix} 0.23 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.72 \end{bmatrix} = \begin{bmatrix} 0.23 \\ 0.77 \end{bmatrix}$$

Cross-Sectional Distribution: Example

- Why go through the pain of this loopy approach rather than writing the definitions of each element of μ_{t+1} individually in the code?
- As you scale-up your problem, you're increasingly likely to make mistakes (e.g. $n\_S = 21$).
- This is easy with the loopy approach.

Cross-Sectional Distribution: Example

- Now let's bring back assets A. Say $a \in \{a_1, a_2\}$.

- Say your household problem solution gives policy functions

$$a'(a, s) = a_2 \ \forall a, s$$

  i.e. they always choose a_2.

- Parameterise with

$$\pi = \begin{bmatrix} 0.50 & 0.50 \\ 0.20 & 0.80 \end{bmatrix}$$

$$\mu_t = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{bmatrix}$$

Cross-Sectional Distribution: Example

- See then that

$$\begin{aligned}\mu_{t+1}(a_1, s_1) &= \mathbb{1}_{a_1=a'(a_1,s_1)}\pi(s_1|s_1)\mu_t(a_1, s_1) \\ &\quad + \mathbb{1}_{a_1=a'(a_1,s_2)}\pi(s_1|s_2)\mu_t(a_1, s_2) \\ &\quad + \mathbb{1}_{a_1=a'(a_2,s_1)}\pi(s_1|s_1)\mu_t(a_2, s_1) \\ &\quad + \mathbb{1}_{a_1=a'(a_2,s_2)}\pi(s_1|s_2)\mu_t(a_2, s_2) \\ &= 0\end{aligned}$$

  since $\mathbb{1}_{a_1=a'(a_2,s_1)} = 0$.

- Expression for $\mu_{t+1}(a_1, s_2) = 0$ similarly.
- I.e. nobody is there since nobody chooses this level of assets.

Cross-Sectional Distribution: Example

- Then for (a_2, s_1)

$$\begin{aligned}
\mu_{t+1}(a_2, s_1) = & \mathbb{1}_{a_2=a'(a_1,s_1)}\pi(s_1|s_1)\mu_t(a_1, s_1) \\
& + \mathbb{1}_{a_2=a'(a_1,s_2)}\pi(s_1|s_2)\mu_t(a_1, s_2) \\
& + \mathbb{1}_{a_2=a'(a_2,s_1)}\pi(s_1|s_1)\mu_t(a_2, s_1) \\
& + \mathbb{1}_{a_2=a'(a_2,s_2)}\pi(s_1|s_2)\mu_t(a_2, s_2)
\end{aligned}$$

$$\begin{aligned}
= & (1) * (0.5) * (0.25) + (1) * (0.2) * (0.25) \\
& + (1) * (0.5) * (0.25) + (1) * (0.2) * (0.25)
\end{aligned}$$

$$= 0.125 + 0.050 + 0.125 + 0.050$$

$$= 0.350$$

Cross-Sectional Distribution: Example

- Then for (a_2, s_2)

$$\begin{aligned}\mu_{t+1}(a_2, s_2) = &\ \mathbb{1}_{a_2=a'(a_1,s_1)}\pi(s_2|s_1)\mu_t(a_1, s_1) \\ &+ \mathbb{1}_{a_2=a'(a_1,s_2)}\pi(s_2|s_2)\mu_t(a_1, s_2) \\ &+ \mathbb{1}_{a_2=a'(a_2,s_1)}\pi(s_2|s_1)\mu_t(a_2, s_1) \\ &+ \mathbb{1}_{a_2=a'(a_2,s_2)}\pi(s_2|s_2)\mu_t(a_2, s_2)\end{aligned}$$

$$\begin{aligned}= &\ (1) * (0.5) * (0.25) + (1) * (0.8) * (0.25) \\ &+ (1) * (0.5) * (0.25) + (1) * (0.8) * (0.25)\end{aligned}$$

$$= 0.125 + 0.200 + 0.125 + 0.200$$

$$= 0.65$$

Cross-Sectional Distribution: Example

- Hence

$$\mu_{t+1} = \begin{bmatrix} 0.00 & 0.00 \\ 0.35 & 0.65 \end{bmatrix}$$

Cross-Sectional Distribution: Recipe for Steady State

- Let's come back to:

```
mup = zeros(n_A, n_S);
 for a_ind = 1:n_A
  for s_ind = 1:n_S
   for sp_ind = 1:n_S
        mup(ap_policy_ind(a_ind, s_ind), sp_ind) = ...
 mup(ap_policy_ind(a_ind, s_ind), sp_ind) + Pi(s_ind, sp_ind)*mu(a_ind, s_ind)
        end
   end
  end
```

- Our example here is $n\_S \; x \; n\_A \; x \; n\_S = 2x2x2$.
- So the nested loops will fill a $2x2$ array for μ_{t+1} in 8 increments.
- Let's go through them, one at at time.

Cross-Sectional Distribution: Example

- $a\_ind = 1$, $s\_ind = 1$, $sp\_ind = 1$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.000 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.125 & 0.000 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.125 & 0.000 \end{bmatrix}$$

- $a\_ind = 1$, $s\_ind = 1$, $sp\_ind = 2$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.125 & 0.000 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.125 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.125 & 0.125 \end{bmatrix}$$

- $a\_ind = 1$, $s\_ind = 2$, $sp\_ind = 1$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.125 & 0.125 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.050 & 0.000 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.175 & 0.125 \end{bmatrix}$$

- $a\_ind = 1$, $s\_ind = 2$, $sp\_ind = 2$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.175 & 0.125 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.200 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.175 & 0.325 \end{bmatrix}$$

Cross-Sectional Distribution: Example

- $a\_ind = 2$, $s\_ind = 1$, $sp\_ind = 1$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.175 & 0.325 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.125 & 0.000 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.300 & 0.325 \end{bmatrix}$$

- $a\_ind = 2$, $s\_ind = 1$, $sp\_ind = 2$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.300 & 0.325 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.125 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.300 & 0.450 \end{bmatrix}$$

- $a\_ind = 2$, $s\_ind = 2$, $sp\_ind = 1$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.300 & 0.450 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.050 & 0.000 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.350 & 0.450 \end{bmatrix}$$

- $a\_ind = 2$, $s\_ind = 2$, $sp\_ind = 2$ gives

$$\mu_{t+1} = \begin{bmatrix} 0.000 & 0.000 \\ 0.350 & 0.450 \end{bmatrix} + \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.200 \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 \\ 0.350 & 0.650 \end{bmatrix}$$

Recursive Competitive Equilibrium Definition

- A steady state recursive competitive equilibrium is a set of policy functions (c, a'), a price q and an invariant cross-section μ such that
  - For the given q, the recursive program solves the household's optimisation problem,
  - Given the stationary cross-section, goods and asset markets clear

$$\int_{\mathcal{S},\mathcal{A}} [c(s,a) - y(s)] d\mu = 0$$
$$\int_{\mathcal{S},\mathcal{A}} [a'(s,a)] d\mu = 0,$$

  - μ is a stationary measure: meaning that plugging μ into the right-side of (1) will return μ on the left-side.

Algorithm for Finding RCE

- Excess demand functions are your best friend!
- Most basic way to solve this:

(a) Conjecture a price q_0,

(b) Given q_0, solve the household's recursive problem. Gives you the corresponding policy functions for controls,

(c) Iterate on the cross-sectional law of motion (1) until it converges,

(d) Find the excess demand for assets. If it's close to zero, stop. If not, update the bond price as follows:

- If excess demand is positive, increase q,
- If excess demand is negative, decrease q.

(e) Return to step (b) using your updated price.

Algorithm for Finding RCE

- In the updating step, (d), you can just use the bisection method!
- E.g. set $q_{ub} = 1$ and $q_{lb} = \epsilon$ and then the initial guess is $q_0 = \frac{q_{ub}+q_{lb}}{2}$.
- If your excess demand is positive, set $q_{lb} = q_0$.
- If your excess demand is negative, set $q_{ub} = q_0$.

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Takeaways

- If you can implement all these techniques from today, you open-up a world of truly exciting research questions that you can answer.
- Get to work!