Lecture 3: Theory of Corporate Finance II
Imperfect and Incomplete Capital Markets

Adam Hal Spencer

The University of Nottingham

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Roadmap

Motivation

- The Modigliani & Miller (1958) derivations we studied last time require frictionless capital markets.
- The idea is that firms pursue their profit-maximisation motive and households can choose their optimal portfolio of securities to suit their own risk appetites.
- What happens when there are frictions preventing free-trading of portfolios and securities in capital markets?

Roadmap

General setup

- A complete capital market is defined as a market, in which a complete set of state contingent claims are priced and traded.
- A state contingent claim is a security that pays 1 unit when the corresponding state of the world prevails in the future and zero otherwise.
- Also referred to sometimes as Arrow-Debreu securities.

Example

- Consider a world with two periods $t \in \{0, 1\}$.
- There is uncertainty surrounding the state of the world at time $t = 1$.
- A representative household seeks to maximise its welfare through consumption across the two time periods.
- They are endowed with y_0 units of consumption good at time $t = 0$ and y_1 at time $t = 1$.
- The amount y_0 is known at time zero, but y_1 is only known to be drawn from a uniform distribution over N possible values over $\{y_1(1), y_1(2), y_1(3), ..., y_1(N)\}$.
- That is: uncertainty for time $t = 1$, but not for time $t = 0$.
- We'll refer to the draw of $y_1(\omega)$ from the feasible set at time $t = 1$ as state ω.

Example

- The market is complete.
- Holdings of state contingent claim for state ω are denoted by $a(\omega)$ while the security price is $\varphi(\omega)$.
- The household consumes in both periods of life and gets utility

$$u(c_0, c_1(\omega)) = \log(c_0) + \beta \mathbb{E}_\omega[c_1(\omega)]$$

  which is their expected utility where the expectation is over the state that prevails next period.
- Characterise the household's optimal consumption-investment plan.

Problem

- The household's problem is given by

$$\max_{c_0,\{a(\omega)\}_{\omega\in 1,2,\dots,N}} \quad \log(c_0) + \beta\mathbb{E}_\omega[\log(c_1(\omega))]$$

subject to the constraints

$$c_0 + \sum_{\omega\in\{1,2,\dots,N\}} a(\omega)\varphi(\omega) = y_0$$

$$c_1(\omega) = y_1(\omega) + a(\omega), \ \ \forall\omega \in \{1, 2, \dots, N\}$$

- The first constraint says the household splits its initial endowment between consumption in $t = 0$ and their portfolio of state contingent claims.

- The second constraint says that, in a given state in time $t = 1$, their consumption is simply made up of the corresponding endowment draw and the payout from their holdings of scc for that state.

Solution

- Substitute the constraints into the objective to get

$$\mathcal{L} = \log\left(y_0 - \sum_{\omega\in\{1,2,\ldots,N\}} a(\omega)\varphi(\omega)\right) + \beta\mathbb{E}_\omega[\log(y_1(\omega) + a(\omega))]$$

meaning that the optimisation problem simplifies-down to a choice of the sccs portfolio.

- Has derivative

$$\frac{\partial \mathcal{L}}{\partial a(\omega)} = \frac{1}{c_0}(-\varphi(\omega)) + \beta\mathbb{E}_\omega\left[\frac{\partial log(c_1(\omega))}{\partial a(\omega)}\right]$$

Solution

- Thus we can write

$$\mathbb{E}_\omega[\log(c_1(\omega))] = \sum_{\omega \in \{1,2,\ldots,N\}} \mathbb{P}\text{rob.}(\omega) \log(c_1(\omega))$$

by the definition of the expectation operator. We can expand the right-hand side as

$$\begin{aligned}\mathbb{E}_\omega[\log(c_1(\omega))] = &\sum_{\omega \in \{1,2,\ldots,N\}} \mathbb{P}\text{rob.}(\omega) \log(c_1(\omega)) \\ =&\mathbb{P}\text{rob.}(1) \log(c_1(1)) + \mathbb{P}\text{rob.}(2) \log(c_1(2)) + \ldots \\ &+ \mathbb{P}\text{rob.}(N) \log(c_1(N))\end{aligned}$$

Solution

- Where see then that when we take the derivative of the expectation

$$\mathbb{E}_\omega\left[\frac{\partial log(c_1(\omega))}{\partial a(\omega)}\right] = \mathbb{P}\text{rob.}(\omega)\frac{\partial \log(c_1(\omega))}{\partial a(\omega)}$$

given that all of the other terms drop-out of the sum (since $a(\omega)$ only appears in $c_1(\omega)$)

Solution

- Thus the solution is characterised by

$$\varphi(\omega) = \beta \mathbb{P}\text{rob.}(\omega) \frac{c_0}{c_1(\omega)} \tag{1}$$

which holds for all $\omega \in \{1, 2, ..., N\}$.

Multiple agents

- What does this mean for consumption when we have multiple agents?
- Say there are two households — A and B. Superscripts will denote the household from now on.
- Say there are two states of the world $\omega \in \{1, 2\}$.
- The income endowments for the households in each state are given by

$$(y_0^A, y_1^A(1), y_1^A(2)) = (1, 1, 0)$$
$$(y_0^B, y_1^B(1), y_1^B(2)) = (1, 0, 1).$$

where each state has a 50-50 chance of occurring.

Multiple agents

- Assume that the state contingent claims are held in zero net supply.
- Means that demand and supply sum to zero.
- Market clearing for state contingent claims is given by

$$a^A(\omega) + a^B(\omega) = 0$$

for $\omega \in \{1, 2\}$. That is — borrowing and lending cancel out, (one person's security holdings are the negative of the other's).

- The two households are lending from/to one another.

Multiple agents

- Our pricing equation (1) for agent $j \in \{A, B\}$ then gives that

$$\varphi(\omega) = \beta \mathbb{P}\text{rob.}(\omega) \frac{c_0^j}{c_1^j(\omega)}$$

$\forall j \in \{A, B\}$ and $\omega \in \{1, 2\}$.

- This means that marginal utilities are equalised across consumers

$$\frac{c_0^A}{c_1^A(\omega)} = \frac{c_0^B}{c_1^B(\omega)}$$

- This is what's known as complete risk sharing.

Multiple agents

- Recall that the agents were both exactly the same except for the states, in which they get the unit payout in $t = 1$.
- Given that the probabilities of each state in $t = 1$ are the same, it follows that everything is equalised across the two households.
- I.e. $c_0^A = c_0^B$, $c_1^A(1) = c_1^B(1)$ and $c_1^A(2) = c_1^B(2)$.

Multiple agents

- Will consumption be equalised across all agents in all states always with market completeness?
- No: preferences could be different, endowments could be different, etc.
- But these transfers between households will all take place such that idiosyncratic risk is mitigated.
- This is a good thing since our households are all risk averse!

Roadmap

What's an Incomplete Market?

- When we do not have a full set of state-contingent claims, the market is said to be incomplete.
- What's the issue here?
- Agent's can no longer perfectly share their idiosyncratic risk.

What's an Incomplete Market?

- Classic example is a riskless bond that delivers net interest of $r > 0$ per period.
- Notice that this interest is not state-contingent!
- The amount of interest the households receive will be the same in the future, regardless of which state arises.

What's an Incomplete Market?

- Let's think about the same setup as the previous section, but now the only asset households can hold are these riskless bonds.

- Household problem is then

$$\max_{c_0, a} \ \log(c_0) + \beta \mathbb{E}_\omega[\log(c_1(\omega))]$$

subject to the constraints

$$c_0 + a = y_0$$
$$c_1(\omega) = y_1(\omega) + a(1 + r), \ \forall \omega \in \{1, 2, ..., N\}$$

What's an Incomplete Market?

- Notice that we still have state-by-state budget constraints for period $t = 1$, but the asset payout is always the same.
- Right now you should be getting suspicious...no dependence of the asset payout on the state is going to make it hard to share risk...

Solution

- Substitute the constraints into the objective to get

$$\mathcal{L} = \log(y_0 - a) + \beta\mathbb{E}_\omega[\log(y_1(\omega) + a(1+r))],$$

which has the derivative

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial a} &= \frac{1}{c_0}(-1) + \beta\mathbb{E}_\omega\left[\frac{\partial log(c_1(\omega))}{\partial a}\right] \\ &= -\frac{1}{c_0} + \beta\mathbb{E}_\omega\left[\frac{1}{c_1(\omega)}(1+r)\right] \\ &= -\frac{1}{c_0} + \beta(1+r)\mathbb{E}_\omega\left[\frac{1}{c_1(\omega)}\right]\end{aligned}$$

where the return comes outside the expectation since it's riskless.

Solution

- The households' Euler equation is then given by

$$1 = \beta(1+r)\mathbb{E}_\omega\left[\frac{c_0}{c_1(\omega)}\right]$$

- What does this mean about risk sharing then?

Solution

- Let's again think about two households: A and B.
- See that the Euler equations imply

$$\mathbb{E}_\omega \left[\frac{c_0^A}{c_1^A(\omega)} \right] = \mathbb{E}_\omega \left[\frac{c_0^B}{c_1^B(\omega)} \right]$$

- How does this differ from the complete markets case?

Solution

- Marginal utilities are now only equalised in in expectation, rather than state-by-state.
- Individual households now face idiosyncratic risk!
- Bad news!

Solution

- The absence of state-contingent claims means households now face risk.
- The asset market setup can greatly impact the welfare of households.

Clientele Effect

- What does this mean from the perspective of firms though?
- With complete markets, households don't really care about the firms' financial policy.
- It's desirable firms to make as much money as possible: maximise the size of the pie for distribution.
- From there, households can just disperse the pie amongst themselves using state contingent claims.
- Value of the firm is just the expected present value of their earnings.

Clientele Effect

- With asset market incompleteness though, the financial policy of a firm can start to impact its value.
- This is known as the clientele effect.
- Firms can increase their value by adjusting their payout/financial policy to cater to the preferences of their clients.

Clientele Effect

- E.g. say there are two states of the world next period (call them boom and bust).
- If the shareholders in the company all have no income in the bust phase, the firm can increase its value by paying-out a lot of dividends during a bust relative to a boom.
- Financial policy can help smooth household consumption in this case.
- Can partially mitigate the impact of a lack of asset market richness.

Roadmap

Trading Frictions

- The term market imperfections is a little more vague.
- It basically refers to the idea that investors can't always make the type of trades that they want.
- Can again influence the value of firms.

Trading Frictions

- E.g. short-selling is when investor C borrows an asset from investor D, sells the asset in the market today and then promises to repay the value of the borrowed asset back to investor D in the future plus interest.
- You do this when the value of the security is perceived to be "too high" today.
- If you expect the price to come-down in the future, you can net a profit from the trade.

Trading Frictions

- China in 2015: temporary ban on short-selling of assets.
- What's the issue here?
- The act of short-selling closes arbitrage opportunities.
- If an asset is over-valued, people will short, short, short until the price comes down and the arbitrage opportunity disappears.

Trading Frictions

- A firm's financial policy might be a determinant of whether its price is too high or low relative to the value of its future cash flows.
- If you ban short-selling, the firm will remain over-valued. The gap never closes!

Roadmap

Takeaways

- Market incompleteness and imperfections can distort trading opportunities.
- Two firms with identical future cash flows may have securities with different values based purely on financial differences.