Lecture 4: Theory of Corporate Finance III
Taxes

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Roadmap

Motivation

- How do investment and capital structure choices change when we introduce corporate taxes?
- Is it better for the firm to issue new debt or equity?

Tax shields

- Interest payments are tax deductible!
- Finance guys refer to this as debt tax shields.
- The more the firm borrows, the more it pays in interest. Thus the more it saves in taxes.
- Can think of this as like getting a cheque back from the government.

Roadmap

Setup

- The difference in this model from lecture 2 is that the firm's corporate earnings are taxable.
- The firm receives the tax shields in $t = 1$ when it repays its debt.
- Lender's problem is the same as in the model without frictions.
- Let's go over the details one more time to make sure it's crystal clear...

Setup in $t = 0$

- Again two time periods $t \in \{0, 1\}$.
- A firm invests in $t = 0$ in productive capital (k).
- It needs to finance this investment by issuing external financing.
- Can issue new debt ($b > 0$) or new equity ($e_0 < 0$).
- Draws a stochastic (random) productivity shock

Setup in $t = 1$

- Draws a stochastic (random) productivity shock (θ) at the start of period $t = 1$.
- This shock is unknown to the firm at time $t = 0$.
- The shock can take one of two values $\theta \in \{0, 1\}$.
- Denote the probability of drawing $\theta = 1$ by $p \in [0, 1]$.
- If the firm has zero productivity the does not produce and thus defaults.
- After they choose to default, the capital stock is handed-over to the creditors, who liquidate it for ξk where $\xi \in [0, 1]$.
- If the firm defaults on its debt, the creditors (lenders) take control of the firm's assets.
- Assume that the capital stock fully depreciates after use.

Setup: Taxes

- Denote the corporate tax rate $\tau \in [0, 1]$.
- The earnings are then given by $(1 - \tau)\theta k^{\alpha}$ when producing.
- The proceeds from liquidating the firm are not taxable.
- The cost of borrowing for the firm (net interest rate) is denoted by r.
- The debt tax shields are given by τbr — the amount of interest is rb — this reduces the firm's overall tax burden.

Roadmap

Firm's problem

- Firm's problem now given by

$$\max_{k,b} v = -k + b + \beta p\{(1-\tau)k^{\alpha} - b(1 + r[1-\tau])\}$$

subject to

$$r = \frac{1}{p}\left[\frac{1}{\beta} - (1-p)\xi\frac{k}{b}\right] - 1$$

where this problem is the same as lecture 2 (frictionless model) except that the tax rate features when producing.

Firm's solution

- Derivative with respect to investment given by

$$
\begin{aligned}
\frac{\partial v}{\partial k} &= -1 + \beta p \left[(1-\tau)\alpha k^{\alpha-1} - b(1-\tau)\frac{\partial r}{\partial k} \right] \\
&= -1 + \beta p \left[(1-\tau)\alpha k^{\alpha-1} + b(1-\tau)\frac{1}{p}(1-p)\xi\frac{1}{b} \right] \\
&= -1 + (1-\tau)\beta p \left[\alpha k^{\alpha-1} + \frac{1-p}{p}\xi \right] \\
&= -1 + (1-\tau)\beta p \alpha k^{\alpha-1} + \beta(1-\tau)(1-p)\xi
\end{aligned}
$$

Firm's solution

- Derivative with respect to debt given by

$$\frac{\partial v}{\partial b} = 1 - \beta p \left[(1 + r[1-\tau]) + b(1-\tau)\frac{\partial r}{\partial b} \right]$$

$$= 1 - \beta p \times$$

$$\left[\left(1 + \left\{ \frac{1}{p}\left[\frac{1}{\beta} - (1-p)\xi\frac{k}{b} \right] - 1 \right\} [1-\tau] \right) + b(1-\tau)\frac{1}{p}(1-p)\xi\frac{k}{b^2} \right]$$

$$= 1 - \beta p - \beta p(1-\tau)\frac{1}{p}\left[\frac{1}{\beta} - (1-p)\xi\frac{k}{b} \right] + \beta p(1-\tau)$$

$$- \beta(1-\tau)(1-p)\xi\frac{k}{b}$$

$$= 1 - \beta p - \beta(1-\tau)\frac{1}{\beta} + \beta(1-\tau)(1-p)\xi\frac{k}{b} +$$

$$\beta p(1-\tau) - \beta(1-\tau)(1-p)\xi\frac{k}{b}$$

$$= \tau(1 - \beta p)$$

Firm's solution

- So $\frac{\partial v}{\partial b} = \tau(1 - \beta p)$.
- What does this mean?
- This number is always positive!
- Borrow as much as you can!
- A unit of extra borrowing gives you the added benefit of the tax deductions.
- Why is there no cost of borrowing more?
- Because the lenders only break even.
- All the benefit is borne by the shareholders.

Borrow as much as you can!

- What are the implications of this solution for the cost of debt?

- If $b \to \infty$ then

$$\lim_{b\to\infty} r = \lim_{b\to\infty} \frac{1}{p}\left[\frac{1}{\beta} - (1-p)\xi\frac{k}{b}\right] - 1$$
$$= \frac{1}{p}\frac{1}{\beta} - 1$$

which is equivalent to finite debt when $\xi = 0$.

- Borrow as much as you can: lender behaves as if there is no liquidation value in the bad state.

- What's the intuition here?

Firm without debt

- Let's again have a look at the problem for the firm without borrowing.
- Recall: here when $\theta = 0$, the shareholders liquidate the firm and get the proceeds (ξk).
- Solves

$$\max_k \hat{v} = -k + \beta\{p(1-\tau)k^\alpha + (1-p)\xi k\}$$

which has derivative

$$\frac{\partial \hat{v}}{\partial k} = -1 + \alpha\beta p(1-\tau)k^{\alpha-1} + \beta(1-p)\xi.$$

- How does this differ from the case with debt? Recall the derivative there was

$$\frac{\partial v}{\partial k} = -1 + (1-\tau)\beta p\alpha k^{\alpha-1} + \beta(1-\tau)(1-p)\xi$$

Firm without debt

- This means that the firm will invest less in the case with debt here?
- How do we interpret that?
- The firm relies more on tax subsidies from the government than sales revenues.
- Why? The last term in the derivative represents the benefit received from more collateral in the case of liquidation.
- That matters less now since the firm is having part of their interest payments subsidised by the government.
- Like the government is paying the difference associated with the higher r in tax rebates.
- Firm is basically exploiting the taxpayer!

Roadmap

Depreciation

- Another common aspect of the tax code is for depreciation to be deductible as well.
- So far we've assumed full depreciation.
- Assume now that $\delta \in [0, 1)$ is the rate of depreciation.
- So some fraction δk of the firm's capital stock is lost after use.
- You can expense this in the amount of $\tau\delta k$.
- Abstract from debt here.

Depreciation

- Firm's problem then becomes

$$\max_k \hat{v} = -k + \beta\{p[(1-\tau)k^\alpha + (1-\delta)k + \tau\delta k] + (1-p)\xi k\}$$

which has derivative

$$\frac{\partial \hat{v}}{\partial k} = -1 + p[\alpha\beta(1-\tau)k^{\alpha-1} + (1-\delta) + \tau\delta] + \beta(1-p)\xi.$$

Without depreciation expense

- How does this differ from when depreciation can not be expensed?
- The marginal benefit of another unit of capital is higher when depreciation can be expensed.
- More investment means big tax rebates from the government.

Roadmap

Summary

- Taxes distort investment incentives.
- When the firm can expense interest, we get this weird scenario where they invest less and make their living from tax rebates!
- Why don't we see this in reality?
- Needs to be some cost associated with borrowing too much!