Lecture 5: Money in the Utility Function

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Roadmap

Recap

- Last time we stuck cash into an RBC model...it didn't work.
- In the sense that there was no money demand as it was a dominated asset in rate of return.
- The government can have any supply any amount they want, but there will never be an equilibrium in the money market.
- Need to give the household some reason for holding money.

Recap

- Fastest and dirtiest way to generate money demand is to stick it in the household's utility function, (like a type of good in the model).
- Sidrauski, M. (1967), "Inflation and Economic Growth", *Journal of Political Economy*, 75, pp. 796 – 810.

Roadmap

Household Setup

- Let's forget about capital for now.
- Assume that households can hold cash m_{t+1} or discount bonds in each period b_{t+1}.
- Otherwise the setup is the same as the RBC model.

Period Utility Function

- Household's period utility function

$$\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(m_{t+1}/p_t)^{1-\nu}}{1-\nu} - \frac{n_t^{1+\psi}}{1+\psi}$$

where m_{t+1}/p_t represents real balances and $\nu > 0$ is a parameter.

- What does this mean?

Period Utility Function

Floyd "Money" Mayweather Jnr.
Former world boxing champion

Period Utility Function

Plaza Bar, Madison WI (USA)
$2.50 Long Island Iced Teas on Thursdays

Household's Problem

- Problem:

$$\max_{\{c_t, n_t, b_{t+1}, m_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(m_{t+1}/p_t)^{1-\nu}}{1-\nu} - \frac{n_t^{1+\psi}}{1+\psi} \right]$$

subject to their budget constraints

$$p_t c_t + q_t b_{t+1} + m_{t+1} \leq w_t n_t + m_t + b_t + d_t$$
$$b_0, m_0 \ \text{given}$$

Firm's Problem

- Problem:

$$\max_{\{n_t\}} d_t = p_t y_t - w_t n_t$$

where the production function is given by

$$y_t = a_t n_t^{\alpha}$$

- Recall that we abstract from capital.
- The firm takes price and wage as given.

Monetary Authority

- Assume that the central bank sets an exogenous supply of money $\{m_t\}_{t=0}^{\infty}$.
- Money law of motion

$$m_{t+1} = e^{\zeta_{t+1}} m_t \tag{1}$$

where

$$\zeta_{t+1} = \rho_\zeta \zeta_t + \epsilon_{\zeta,t+1}, \;\; \epsilon_{\zeta,t+1} \sim N(0, \sigma_\zeta^2). \tag{2}$$

- Can be re-written in terms of real balances as

$$\frac{m_{t+1}}{p_t} = e^{\zeta_{t+1}} \frac{m_t}{p_{t-1}} \frac{1}{\pi_t}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation rate at t.

Roadmap

Household Optimality

- Lagrangian given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(m_{t+1}/p_t)^{1-\nu}}{1-\nu} - \frac{n_t^{1+\psi}}{1+\psi} \right] + \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t \left[w_t n_t + m_t + b_t + d_t - m_{t+1} - q_t b_{t+1} - p_t c_t \right]$$

Household Optimality: First Order Conditions

- FOCs:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t c_t^{-\sigma} - p_t \lambda_t = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \Rightarrow -\beta^t n_t^{\psi} + \lambda_t w_t = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = 0 \Rightarrow -q_t \lambda_t + \mathbb{E}_t[\lambda_{t+1}] = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial m_{t+1}} = 0 \Rightarrow -\lambda_t + \mathbb{E}_t[\lambda_{t+1}] + \beta^t \frac{1}{p_t} \left(\frac{m_{t+1}}{p_t} \right)^{-\nu} = 0 \tag{6}$$

where notice there is an additional benefit term in the FOC for m_{t+1} relative to last class.

Firm Optimality: First Order Conditions

- FOC

$$\frac{\partial d_t}{\partial n_t} = 0 \Rightarrow \alpha p_t a_t n_t^{\alpha-1} - w_t = 0 \tag{7}$$

Equilibrium Definition

- The equilibrium of the MIU model is defined as a sequence of prices $\{w_t, p_t, q_t\}_{t=0}^{\infty}$ and allocations $\{c_t, m_{t+1}, b_{t+1}, n_t\}$ with the state vector $\{m_t, a_t, b_t\}$ taken as given by the agents in the model. Optimality conditions (3) – (7) above hold, the household's budget constraint binds and all markets clear.

Canonical Representation

- Money demand from (6), (5) and (3)

$$\mu_{t+1}^{-\nu} c_t^{\sigma} = 1 - q_t$$

where $\mu_{t+1} = m_{t+1}/p_t$ is real balances at t.

- Labour supply (how does this differ from last class?)

$$c_t^{\sigma} n_t^{\psi} = \omega_t$$

where $\omega_t = w_t/p_t$ is the real wage at t.

- Labour demand

$$\alpha a_t n_t^{\alpha-1} = \omega_t$$

Canonical Representation

- Euler equation (how does this differ from last class?)

$$q_t = \mathbb{E}_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right]$$

where $\pi_{t+1} = p_{t+1}/p_t$ is the gross inflation rate at $t+1$.

- Money supply

$$\frac{\mu_{t+1}}{\mu_t} = e^{\zeta_{t+1}} \frac{1}{\pi_t} \tag{8}$$

- Budget constraint (where is this coming from?)

$$\omega_t n_t + \mu_t \frac{1}{\pi_t} = \mu_{t+1} + c_t$$

- Six equations in six unknowns $\{\mu_{t+1}, c_t, q_t, n_t, \omega_t, \pi_t\}$.

Big Picture: What Affects Money Supply and Demand?

- Money supply

$$\mu_{t+1} = \mu_t e^{\zeta_{t+1}} \frac{1}{\pi_t}$$

- Money demand

$$\mu_{t+1} = \left(\frac{1}{1 - q_t} \right)^{\frac{1}{\nu}} c_t^{\frac{\sigma}{\nu}}$$

- Gross nominal interest rate versus bond price

$$i_t = \frac{1}{q_t}$$

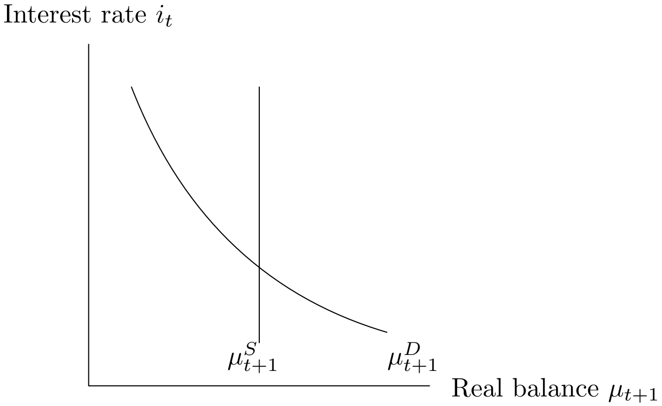
Big Picture: What Affects Money Supply and Demand?

- Money demand in terms of nominal interest rate

$$\mu_{t+1} = \left(\frac{i_t}{i_t - 1} \right)^{\frac{1}{\nu}} c_t^{\frac{\sigma}{\nu}}$$

which is decreasing in i_t for $\nu > 0$.

Big Picture: What Affects Money Supply and Demand?



Big Picture: What Affects Money Supply and Demand?

- This figure looks a lot like our money demand equilibrium from the Keynesian framework.
- Changes in the money supply shift the vertical line.
- Equilibrium settles with a new nominal interest rate.
- E.g. decrease the real supply of cash, leads to an increase in the nominal rate.
- What happens to the real economy afterwards?

Roadmap

What Does the Steady State Look Like?

- Steady state money supply equation (8)

$$\frac{\bar{\mu}}{\bar{\mu}} = e^0 \frac{1}{\bar{\pi}}$$
$$\Rightarrow \bar{\pi} = 1$$

given that steady state has shocks shut-down ($\zeta_{t+1} = 0$).

- What is the implication of $\bar{\pi} = 1$?

What Does the Steady State Look Like?

- Steady state Euler equation

$$\bar{q} = \beta \frac{1}{\bar{\pi}}$$

- Steady state labour supply

$$\bar{c}^{\sigma} \bar{n}^{\psi} = \bar{\omega}$$

- Steady state labour demand

$$\alpha \bar{n}^{\alpha - 1} = \bar{\omega}$$

What Does the Steady State Look Like?

- Steady state money demand

$$\bar{\mu} = \left(\frac{\bar{i}}{\bar{i}-1}\right)^{\frac{1}{\nu}} \bar{c}^{\frac{\sigma}{\nu}}$$

- Steady state budget constraint

$$\bar{\omega}\bar{n} = \bar{c}$$

Steady State Money Neutrality

- Nothing other than nominal variables and real balances depend on money!
- Money is still neutral!

Steady State Money Neutrality

- Changes in the money supply will affect the nominal interest rate.
- But since there is no price rigidity in this model, this won't spill-over to affect the real interest rate as it would in our IS-LM model from the Keynesian theory.
- A change in the money market will not shift the LM curve in this context.
- Since LM is in gdp-real interest rate space and no impact on the real rate since no price rigidity.

Roadmap

Log-Linearisation

- Log-linearised labour supply

$$\sigma \hat{c}_t + \psi \hat{n}_t = \hat{\omega}_t$$

- Log-linearised labour demand

$$\hat{a}_t + (\alpha - 1)\hat{n}_t = \hat{\omega}_t$$

Log-Linearisation

- Log-linearised Euler equation

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] + \frac{1}{\sigma}(\hat{q}_t + \mathbb{E}_t[\hat{\pi}_{t+1}])$$

Recall that $q_t = 1/i_t \Rightarrow \hat{q}_t = -\hat{i}_t$.

$$\Rightarrow \hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] + \frac{1}{\sigma}(-\hat{i}_t + \mathbb{E}_t[\hat{\pi}_{t+1}])$$

Recall the Fisher equation

$$i_t = r_t \mathbb{E}_t[\pi_{t+1}]$$

Implies that

$$\hat{i}_t = \hat{r}_t + \mathbb{E}_t[\hat{\pi}_{t+1}]$$

$$\Rightarrow \hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \frac{1}{\sigma}\hat{r}_t$$

Log-Linearisation

- Log-linearised money demand (exercise, see appendix for derivation)

$$\hat{\mu}_{t+1} = \frac{\sigma}{\nu}\hat{c}_t - \left\{\frac{1}{\nu(\bar{i}-1)}\right\}\hat{i}_t$$

does it seem right? Increasing in $\hat{c}_t$ and decreasing in i_t?

- Log-linearised budget constraint

$$\bar{c}\hat{c}_t + \bar{\mu}\hat{\mu}_{t+1} = \bar{c}(\hat{\omega}_t + \hat{n}_t) + \bar{\mu}(\hat{\mu}_t - \hat{\pi}_t)$$

- Log-linearised money supply

$$\hat{\mu}_{t+1} = \hat{\mu}_t - \hat{\pi}_t + \zeta_{t+1}$$

Punch-Line: Short-Run Dynamics

- Deviations in nominal variables and real variables (excluding balances) are all separate. Why?
- Again, money is neutral!

Roadmap

Neutrality in the MIU Model

- Why does this happen?
- It's due to the separability of real balances from consumption and labour supply in the household utility function.
- It's like saying, money is another commodity that the households like to consume.

Non-Separable Utility

- This result breaks when we remove this separability feature.
- For example when the utility function is given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right\}$$

where X_t is a composite index over consumption and real balances

$$X_t \equiv \left[(1-\omega) C_t^{1-\nu} + \omega \left(\frac{M_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

for $\nu \neq 1$ and $\omega \in [0,1]$ is a weight.

Non-Separable Utility

- When we take FOCs for the non-separable utility function, suddenly the marginal utility of real balances interacts with consumption and labour supply.
- The amount of cash you have can impact how much you dislike working.
- You can end-up with a scenario where the economy's output depends on its nominal interest rate through this channel.
- Usually the quantitative impact is relatively small though.

Non-Separable Utility

- How does all this contrast with the old Keynesian ideas surrounding non-neutrality?
- There we got non-neutrality because of price rigidities. Changes in the supply would affect the real return through the Fisher equation.
- In the MIU with non-separable preferences: can still get the real interest rate being affected, but rather through money's effect on preferences.

Roadmap

Social Planner's Problem

- Optimal allocation achieved through maximising welfare subject only to technology (just like last class).

- Social planner's problem is

$$\max_{\{c_t, n_t, \mu_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(\mu_{t+1})^{1-\nu}}{1-\nu} - \frac{n_t^{1+\psi}}{1+\psi} \right]$$

  subject to technology $c_t = a_t n_t^{1-\alpha}$.

- No technology with regard to real balances.

- Effectively assumes that it's costless for the government to print money.

Social Planner's Problem: Optimality

- Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(\mu_{t+1})^{1-\nu}}{1-\nu} - \frac{n_t^{1+\psi}}{1+\psi} \right] + \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t [a_t n_t^{1-\alpha} - c_t]$$

- FOCs

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t c_t^{-\sigma} - \lambda_t = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \Rightarrow -\beta^t n_t^{\psi} + \lambda_t (1-\alpha) a_t n_t^{-\alpha} = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{t+1}} = 0 \Rightarrow \beta^t \mu_{t+1}^{-\nu} = 0 \tag{11}$$

Friedman Rule

- How do we achieve (11) in the decentralised setting?
- Recall that $\mu_{t+1}^{-\nu} c_t^{\sigma} = 1 - q_t$ in decentralised setting.
- To get $\mu_{t+1}^{-\nu} = 0$, we need for $1 - q_t = 0$.
- That is: if $q_t = 1$, (no discount on bonds).
- Thus $i_t = 1$ (no net interest).
- Called the Friedman rule.

Friedman Rule

- Social marginal cost of creating extra money is zero.
- Private marginal cost is zero when there is a zero nominal interest rate.
- When $i_t = 1$, see from the Fisher equation that

$$\mathbb{E}_t[\pi_{t+1}] = \frac{1}{r_t}$$

- Those who hold money suffer no losses in its value due to inflation.

Friedman Rule

- Recall the issues we had prior to putting money in the utility function.
- Nobody would hold cash unless the net return to bonds is zero.
- This is the same story here.
- If the net return on bonds is zero, then nobody feels bad about holding cash.
- Since it implies that the two are perfect substitutes.

Roadmap

Takeaways

- MIU model is a first attempt at creating a market for money.
- Cheap as it just changes preferences.
- Optimal policy is the Friedman rule.

Log-Linearisation of Complicated Function Tricks

- Say we want to liniearise $y_t = f(x_t)$. See that

$$
\begin{aligned}
f(x_t) &\approx f(\bar{x}) + f'(\bar{x})(x_t - \bar{x}) \\
\Rightarrow f(x_t) - f(\bar{x}) &= f'(\bar{x})(x_t - \bar{x}) \\
\Rightarrow \frac{f(x_t) - f(\bar{x})}{f(\bar{x})} &= \frac{f'(\bar{x})}{f(\bar{x})}(x_t - \bar{x}) \\
\Rightarrow \hat{y}_t &= \frac{f'(\bar{x})}{f(\bar{x})}\bar{x}\hat{x}_t
\end{aligned}
$$

Log-Linearisation of Complicated Function Tricks

- Now see that $\mu_{t+1} = u_t c_t^{\sigma/\nu}$ where $u_t = \left(\frac{i_t}{i_t - 1}\right)^{\frac{1}{\nu}}$ for money demand.

- See that $\hat{u}_t = -\frac{1}{\nu(\bar{i}-1)}\hat{i}_t$.

- For general function $z_t = f(x_t, y_t)$, see that (exercise, show it)

$$\bar{z}\hat{z}_t = f_x(\bar{x}, \bar{y})\bar{x}\hat{x} + f_y(\bar{x}, \bar{y})\bar{y}\hat{y}$$

  Applying to the money demand gives

$$\hat{\mu}_{t+1} = \frac{1}{\bar{\mu}}\bar{c}^{\frac{\sigma}{\nu}}\left(\frac{\bar{i}}{\bar{i}-1}\right)^{\frac{1}{\nu}}\left[\frac{\sigma}{\nu}\hat{c}_t - \left\{\frac{1}{\nu(\bar{i}-1)}\right\}\hat{i}_t\right]$$

  Then use the steady state relationship to get the final expression.