Lecture 5: Theory of Corporate Finance IV
Costly Bankruptcy

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Roadmap

1. Introduction
2. Model Environment
3. Model Equilibrium
4. Conclusion

Motivation

- Have we thought about bankruptcy in our analysis so far?
- Yes! When the firm defaults, it hands-over the firm's capital to the creditors.
- They liquidate it and that's that.
- But here, bankruptcy didn't come at a cost.
- It's usually an expensive procedure that involves legal fees and a lot of drama.

Motivation

- When we thought about debt tax shields, we saw that if that was the only friction, then the firm should borrow as much as they can.
- What happens now if there is some cost of bankruptcy, which depends on the level of the firm's debt?
- It will give us an interior solution for borrowing!
- What we observe in the real world. Makes a lot more sense.

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Setup

- Abstract from taxes for now, (we'll add them back in later).
- If the firm defaults, the creditors incur legal expenses.
- Why don't the debtors pay these expenses?
- Because they have limited liability. They're underwater anyway, so they just walk away from the situation.
- The creditors need to pay the bankruptcy costs to recover some of their funds.

Setup

- In the event of bankruptcy, assume that a cost function of the following form is incurred:

$$\Omega(b) = \omega b^2.$$

- Says that the cost is increasing and convex in the amount the firm borrows.
- Does this make sense? Says that the more the firm borrows, the more expensive the bankruptcy legal fees are. These fees are increasing at an increasing rate.

Setup

- Again, work with the same model from lecture 2.
- We'll again think about the two period model with $\theta \in \{0, 1\}$ for productivity and production function θk^{α}.
- Firm produces when $\theta = 1$ and defaults for $\theta = 0$ with ξk being the proceeds from liquidating the firm.
- Firm borrows an amount b with endogenous interest rate r set by lenders such that the lenders break even.

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Lender's problem

- The lender's problem changes to account for the fact that these costs are incurred in the event of bankruptcy.

- The lender demands interest rate r such that

$$-b + \beta\{pb(1+r) + (1-p)[\xi k - \Omega(b)]\} = 0$$
$$-b + \beta\{pb(1+r) + (1-p)[\xi k - \omega b^2\} = 0$$

- See that the creditors still break even, but now there is this loss incurred in the default state.

- Intuitively, without doing any math, what does this mean with regard to the interest rate that they demand?

- Higher: to compensate for bigger losses in the default state.

Lender's problem

- Can solve for the interest rate demanded as (divide through by b)

$$
\begin{aligned}
& -1+\beta\left\{p(1+r)+(1-p)\left[\xi\frac{k}{b}-\omega b\right]\right\}=0 \\
\Rightarrow r = & \frac{1}{p}\left[\frac{1}{\beta}-(1-p)\left\{\xi\frac{k}{b}-\omega b\right\}\right]-1
\end{aligned}
$$

does this make sense?

- Increase the cost scaling parameter ω, leads to a rise in the interest rate demanded.

Ex-ante v.s. ex-post

- This is an interesting result.
- Although the creditors incur the default costs in the event of bankruptcy, they pass this cost on to the firm in expectation through a higher interest rate.
- Creditors bear the default costs ex-post.
- Debtors bear the default costs ex-ante.
- How does this then affect the firm's problem?

Firm's problem

- Firm's problem then given by

$$\max_{\{b,k\}} v = -k + b + \beta p[k^{\alpha} - b(1+r)]$$

subject to

$$r = \frac{1}{p}\left[\frac{1}{\beta} - (1-p)\left\{\xi\frac{k}{b} - \omega b\right\}\right] - 1$$

which means that $\frac{\partial r}{\partial b}$ will change!

- An increase in the firm's borrowings now has a larger effect on their borrowing costs.

Firm's solution

- Derivative with respect to investment given by

$$
\begin{aligned}
\frac{\partial v}{\partial k} &= -1 + \beta p \left[\alpha k^{\alpha-1} - b \frac{\partial r}{\partial k} \right] \\
&= -1 + \alpha\beta p k^{\alpha-1} + b\beta p \frac{1}{p}(1-p)\xi \frac{1}{b} \\
&= -1 + \alpha\beta p k^{\alpha-1} + \beta(1-p)\xi
\end{aligned}
$$

...unaffected.

Firm's solution

- Derivative with respect to debt

$$\begin{aligned}
\frac{\partial v}{\partial b} &= 1 - \beta p \left[(1+r) + b \frac{\partial r}{\partial b} \right] \\
&= 1 - \beta p \left[\frac{1}{p} \left\{ \frac{1}{\beta} - (1-p) \left\{ \xi \frac{k}{b} - \omega b \right\} \right\} + b \left\{ \frac{1-p}{p} \left(\xi \frac{k}{b^2} + \omega \right) \right\} \right] \\
&= 1 - \beta \left\{ \frac{1}{\beta} - (1-p) \left\{ \xi \frac{k}{b} - \omega b \right\} \right\} - \beta b (1-p) \left(\xi \frac{k}{b^2} + \omega \right) \\
&= 1 - 1 + \beta (1-p) \left\{ \xi \frac{k}{b} - \omega b \right\} - \beta (1-p) \left[\xi \frac{k}{b} + \omega b \right] \\
&= -\beta (1-p) \omega b - \beta (1-p) \omega b \\
&= -2 \beta (1-p) \omega b
\end{aligned}$$

does this make sense?

- The derivative is always negative for $b > 0$.

Firm's solution

- When the only present financial friction is costly default, then the firm won't take any debt.
- There's only a cost in this case: no advantage.
- Opposite problem to the taxes lecture.
- We need a reasonable theory of optimal borrowing!

Trade-off theory

- When we include both costly default and debt tax shields, we get what the finance guys refer to as the trade-off theory.
- When we include each friction separately we get weird results: either maximal debt or none.
- Include both frictions at the same time: gives us an interior solution.

Trade-off theory: firm's problem

- Firm's problem now given by

$$\max_{k,b} v = -k + b + \beta p\{(1-\tau)k^{\alpha} - b(1 + r[1-\tau])\}$$

subject to

$$r = \frac{1}{p}\left[\frac{1}{\beta} - (1-p)\left\{\xi\frac{k}{b} - \omega b\right\}\right] - 1$$

Trade-off theory: firm's solution

- Investment derivative is the same as in the case with debt tax shields.
- Debt derivative is now

$$\begin{aligned}\frac{\partial v}{\partial b} =& 1-\beta p\left[(1+r[1-\tau])+b(1-\tau)\frac{\partial r}{\partial b}\right]\\ =& 1-\beta p\left(1+\left\{\frac{1}{p}\left[\frac{1}{\beta}-(1-p)\left(\xi\frac{k}{b}-\omega b\right)\right]-1\right\}[1-\tau]\right)\\ &-\beta p\left\{b(1-\tau)\left[\frac{1}{p}(1-p)\left(\xi\frac{k}{b^2}+\omega\right)\right]\right\}\\ =&\tau(1-\beta p)-2\beta(1-\tau)(1-p)\omega b\end{aligned}$$

Trade-off theory: firm's solution

- The optimal debt choice involves setting this derivative equal to zero and solving for b

$$\tau(1-\beta p) - 2\beta(1-\tau)(1-p)\omega b = 0$$
$$\Rightarrow b = \frac{\tau(1-\beta p)}{2\beta(1-\tau)(1-p)\omega}$$

does this make sense?

- We've found an interior solution for borrowing!

- Consistent with the data: firms hold some intermediate level of debt, not infinite or zero only.

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Summary

- Introduced a cost in the case of default.
- When combined with debt tax shields, this gives an interior solution for borrowings!