Lecture 6: Cash in Advance

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Roadmap

Recap

- Last time we introduced money into the RBC model.
- Gave us some insights, but this approach is pretty cheap.
- CIA: assume that money is required to undertake certain types of transactions.
- Without money, these transactions cannot be made.
- Medium of exchange.
- Clower, R., (1967), "A Reconsideration of the Microfoundations of Monetary Theory", *Economic Inquiry*, 6, pp. 1–8.

When Cash is Mandatory for Transactions...

U.S. credit/debit cards don't work in Cuba...

Roadmap

Binding and Slack Constraints

- Until now we've always considered Lagrangians of the form

$$\mathcal{L} = \sum_{t=0}^{\infty} f(x_t) + \sum_{t=0}^{\infty} \lambda_t [\bar{g} - g(x_t)]$$

  where $f(x_t)$ is our objective and our constraint says $g(x_t) \leq \bar{g}$.

- Since the constraint was usually a budget constraint and utility is increasing in consumption, we'd always have that $g(x_t) = \bar{g}$, (i.e. the constraint binds).

- What happens more generally though if $g(x_t) < \bar{g}$?...

Binding Constraints

- Notice that when the constraint binds $\forall t$, the term

$$\sum_{t=0}^{\infty} \lambda_t [\bar{g} - g(x_t)] = 0$$

given that $\bar{g} = g(x_t)$.

- Then we'd have that $\lambda_t > 0$ and $\mathcal{L}$ is equal to the optimised objective.

Slack v.s. Binding Constraints

- If $g(x_t) < \bar{g}$ then to have $\mathcal{L}$ equal to the maximised objective at the solution, we'd need for $\lambda_t = 0$.
- Whenever the constraint is slack, $\lambda_t = 0$.
- Whenever the constraint binds, $\lambda_t > 0$.

Roadmap

Household Setup

- Households derive utility from consumption.
- Abstract from labour supply here.
- Three types of assets are held: cash, capital and riskless one period bonds.
- I.e. households own the capital stock here.
- Assume simple log utility over consumption each period.
- Deterministic model: abstract from any random shocks.
- Each period the household receives a lump-sum cash transfer from the government.

Goods

- Two types of goods: *cash* and *credit*.
- Cash goods are subject to the CIA constraint.
- Today we'll assume that consumption goods are for cash and capital are credit goods.

Timing

(1) Household enters time period t with state vector (m_t, b_t, k_t) of cash, bonds and capital respectively.

(2) Firms produce and goods market trades take place.

(3) Asset market opens and trades take place.

(4) Household leaves period t with state $(m_{t+1}, b_{t+1}, k_{t+1})$

Household Problem

- Household solves the problem

$$\max_{\{m_{t+1},b_{t+1},k_{t+1},c_t\}} \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to

$$\begin{aligned} p_t c_t \leq & m_t + t_t \\ m_{t+1} + b_{t+1} + p_t(k_{t+1} - (1-\delta)k_t) \leq & m_t + t_t - p_t c_t \\ & + b_t(i_t) + \epsilon_t k_t + p_t d_t \end{aligned}$$

where the first constraint is the CIA constraint and the second is the budget constraint.

- Why is there a price on investment?
- i_t here denotes interest on riskless bonds maturing at time t.

Household Problem

- If there is no price on a variable, then it's denoted in terms of cash.
- E.g. m_{t+1} and t_t.
- If there's a price on a variable, then it's delivered in units of the corresponding good.
- E.g. c_t and investment both come from final goods produced.

Household Problem

- CIA constraint in real terms

$$c_t \leq \frac{\mu_t}{\pi_t} + \tau_t$$

where $\mu_t = \frac{m_t}{p_{t-1}}$, $\pi_t = \frac{p_t}{p_{t-1}}$ and $\tau_t = \frac{t_t}{p_t}$.

- Budget constraint in real terms

$$\begin{aligned} \mu_{t+1} + \gamma_{t+1} + k_{t+1} - (1-\delta)k_t \leq & \frac{\mu_t}{\pi_t} + \tau_t - c_t \\ & + \frac{\gamma_t}{\pi_t}(i_t) + \iota_t k_t + d_t \end{aligned}$$

where $\gamma_{t+1} = \frac{b_{t+1}}{p_t}$ and $\iota_t = \frac{\epsilon_t}{p_t}$.

Firm Problem

- Firm solves the problem

$$\max_{\{k_{t+1}\}} p_t d_t = p_t k_t^{\alpha} - \epsilon_t k_t$$

Monetary Authority

- Money supply for the period equal to that from last plus the additional needed to cover the transfers

$$m_{t+1} = m_t + t_t$$

- Assume further that $t_t = gm_t$ for $g > 0$ for simplicity.

- Then

$$m_{t+1} = (1 + g)m_t$$

meaning that the money supply grows at a constant rate.

Roadmap

Household Optimality

- Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \log(c_t) + \sum_{t=0}^{\infty} \lambda_{m,t} \left[\frac{\mu_t}{\pi_t} + \tau_t - c_t \right] + \sum_{t=0}^{\infty} \lambda_{a,t} \times$$
$$\left[\frac{\mu_t}{\pi_t} + \tau_t - c_t + \frac{\gamma_t}{\pi_t}(i_t) + \iota_t k_t + d_t - \mu_{t+1} - \gamma_{t+1} - k_{t+1} + (1-\delta)k_t \right]$$

Household Optimality: FOCs

- FOCs

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t \frac{1}{c_t} - \lambda_{m,t} - \lambda_{a,t} = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{t+1}} = 0 \Rightarrow -\lambda_{a,t} + \lambda_{m,t+1} \frac{1}{\pi_{t+1}} + \lambda_{a,t+1} \frac{1}{\pi_{t+1}} = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow -\lambda_{a,t} + \lambda_{a,t+1} [\iota_{t+1} + (1 - \delta)] = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma_{t+1}} = 0 \Rightarrow -\lambda_{a,t} + \lambda_{a,t+1} \frac{i_{t+1}}{\pi_{t+1}} = 0 \tag{4}$$

Firm Optimality: FOCs

- FOCs

$$\frac{\partial p_t d_t}{\partial k_t} = 0 \Rightarrow \alpha p_t k_t^{\alpha-1} - \epsilon_t = 0 \quad (5)$$
$$\Rightarrow \iota_t = \alpha k_t^{\alpha-1}$$

Canonical Representation

- From (1), see that

$$\lambda_{a,t} + \lambda_{m,t} = \beta^t \frac{1}{c_t}.$$

- Then (2) gives

$$\lambda_{a,t} = \frac{1}{\pi_{t+1}} \{\lambda_{a,t+1} + \lambda_{m,t+1}\}$$

- Using both then gives that

$$\lambda_{a,t+1} + \lambda_{m,t+1} = \beta^{t+1} \frac{1}{c_{t+1}}.$$

Canonical Representation

- Then money demand is given by

$$\lambda_{a,t} = \beta^{t+1} \frac{1}{\pi_{t+1} c_{t+1}} \tag{6}$$

where $\lambda_{a,t} > 0$ means money is valued. Why?

- Need cash for consumption.
- Right-side is next periods marginal utility of consumption discounted by the inflation rate.
- This is what matters when deciding on how much cash to take with you into $t+1$.

Canonical Representation

- (3) and (4) give a no arbitrage condition

$$\iota_{t+1} + (1 - \delta) = \frac{i_{t+1}}{\pi_{t+1}} \quad (7)$$

which says the return on capital equals the real return on bonds. What happens if this doesn't hold?

Canonical Representation

- Euler equation

$$\beta^t \frac{1}{c_t} - \lambda_{m,t} = \left\{ \beta^{t+1} \frac{1}{c_{t+1}} - \lambda_{m,t+1} \right\} [\iota_{t+1} + (1 - \delta)] \tag{8}$$

- If the CIA constraint is slack, this is our standard Euler equation.
- The presence of this CIA constraint distorts the consumption Euler equation.

Canonical Representation

- Bond demand from (4) and (2)

$$i_{t+1} = \frac{\lambda_{m,t+1} + \lambda_{a,t+1}}{\lambda_{a,t+1}} \tag{9}$$

- Resource constraint

$$c_t + k_{t+1} - (1-\delta)k_t = k_t^{\alpha} \tag{10}$$

- Money growth rule

$$\mu_{t+1} = (1+g)\frac{\mu_t}{\pi_t} \tag{11}$$

Equilibrium Definition

- The equilibrium of the CIA model is a sequence $\{c_t, k_t, b_t, m_t, \tau_t\}_{t=0}^{\infty}$ and a sequence of prices $\{p_t, \epsilon_t, i_{t+1}\}_{t=0}^{\infty}$ such that the household optimises and markets clear.

Roadmap

Binding v.s. Non-Binding

- See from (9) that $i_{t+1} > 1 \iff \lambda_{m,t+1} > 0$.
- This means that the CIA constraint binds iff the net nominal interest rate is positive.
- If the opportunity cost of holding money is positive, we'll only hold enough to facilitate our purchases and no more.
- If $i_{t+1} = 1$ then the constraint is slack. Why?

Binding v.s. Non-Binding

- From the Fisher equation, see that

$$r_t = \frac{i_{t+1}}{\pi_{t+1}} \Rightarrow i_{t+1} = r_t \pi_{t+1} \tag{12}$$

- Using (9) and (12) gives that

$$\begin{aligned} \frac{\lambda_{m,t+1} + \lambda_{a,t+1}}{\lambda_{a,t+1}} &= r_t \pi_{t+1} \\ &\Rightarrow \lambda_{m,t+1} = (r_t \pi_{t+1} - 1)\lambda_{a,t+1} \end{aligned} \tag{13}$$

which says that $\lambda_{m,t+1} > 0$ iff $r_t \pi_{t+1} > 1$. Implies $r_t > \frac{1}{\pi_{t+1}}$.

Binding v.s. Non-Binding

- What's the real return on holding cash?
- Recall the Fisher equation $r = i/\pi$.
- The gross return on cash is 1, (meaning no net return).
- Gives the real rate of return of $1/\pi$.
- So the CIA constraint binds when the real return on bonds dominates the real return on cash.
- Again, all about opportunity cost of holding cash.

Roadmap

Short v.s. Long Run in this Model

- There are no deviations once we reach steady state. Why?

What Does the Steady State Look Like?

- We'll get a steady state in real variables.
- What about nominal variables? E.g. m_t or p_t?
- Will the multipliers be constants? Why?

What Does the Steady State Look Like?

- See that (11) implies

$$\bar{\pi} = (1 + g)$$

the gross inflation rate equals the gross growth rate of money.

- We next look at two cases: (i) CIA constraint binds in ss and (ii) CIA constraint is slack in ss.

What Does the Steady State Look Like? Slack CIA

- Implies $\bar{i} = 1$.
- Equation (8) gives that (using the multiplier equalling zero)

$$\frac{1}{\beta} = [\bar{\iota} + (1 - \delta)].$$

- Also see that (7) gives

$$\frac{1}{1+g} = [\bar{\iota} + (1 - \delta)].$$

- We can only have steady state here if $\beta = 1 + g$.

What Does the Steady State Look Like? Slack CIA

- But $\beta < 1$ since it's a discount factor, (discount the future due to preference for more immediate consumption).
- What does this mean for g and $\bar{\pi}$?
- Steady state exists when $g = 1 - \beta < 0$.
- Negative growth in the money supply.
- Negative inflation.
- Rather than the value of money falling over time, it's increasing. Positive real rate of return.
- People happy to hold an abundance of cash since it gives them a real return: no need to hold "just enough" to make their purchases.

What Does the Steady State Look Like? Binding CIA

- Implies $\bar{i} > 1$.
- Equation (6) yields

$$\lambda_t^a = \beta^{t+1} \frac{1}{\bar{\pi}\bar{c}} \tag{14}$$

What Does the Steady State Look Like? Binding CIA

- See from equation (9) that

$$\lambda_{m,t+1} = \lambda_{a,t+1}[\bar{i} - 1] \tag{15}$$

- Then also notice that the FOC for bonds (4) says that

$$\begin{aligned} \frac{\bar{i}}{\bar{\pi}} &= \frac{\lambda_{a,t}}{\lambda_{a,t+1}} \\ \Rightarrow \bar{i} &= \beta^{-1}\bar{\pi} \end{aligned} \tag{16}$$

where the second line follows from using (14).

What Does the Steady State Look Like? Binding CIA

- Equation (16) in (15) gives

$$\begin{aligned}\lambda_{m,t+1} &= \lambda_{a,t+1}[\beta^{-1}\bar{\pi} - 1] \\ &= \beta^{t+2}\frac{1}{\bar{\pi}\bar{c}}[\beta^{-1}\bar{\pi} - 1] \\ &= \beta^{t+1}\frac{1}{\bar{c}}\left(1 - \frac{\beta}{\bar{\pi}}\right)\end{aligned}$$

which also means that $\lambda_{m,t} = \beta^t \frac{1}{\bar{c}}\left(1 - \frac{\beta}{\bar{\pi}}\right)$ since it must hold $\forall t$.

What Does the Steady State Look Like? Binding CIA

- Equations (8) and these expressions for the $\lambda_{m,t}$ and $\lambda_{m,t+1}$ gives

$$\beta^t \frac{1}{\bar{c}} - \beta^t \frac{1}{\bar{c}} \left\{ 1 - \frac{\beta}{\bar{\pi}} \right\}$$
$$= \left\{ \beta^{t+1} \frac{1}{\bar{c}} - \beta^{t+1} \frac{1}{\bar{c}} \left\{ 1 - \frac{\beta}{\bar{\pi}} \right\} \right\} [\bar{\iota} + (1 - \delta)]$$
$$\Rightarrow 1 = \beta[\bar{\iota} + (1 - \delta)]$$

which means that the capital stock is independent of nominal variables!

Effect of Money on Capital

- We don't need cash to buy capital.
- If we put capital goods into the CIA constraint, this independence result would break.
- **Punchline:** that's the problem with CIA, the results are greatly impacted by our assumptions on what goods require cash.

Roadmap

Summary

- CIA model seems a little more natural than MIU.
- Results can be similar depending on assumptions.
- The CIA results are fragile to the assumptions you make: unappealing theoretically.