

FIN 325 Corporate Finance

L5 (Theory): Modigliani and Millter Theory of Capital Structure

Instructor: Adam Hal Spencer¹

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¹Departments of Economics and Finance, UW–Madison.

What is capital structure?

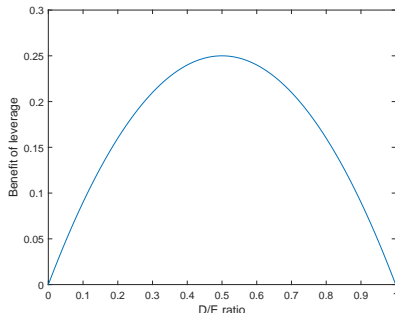
- So far in the course, we've focused on figuring out which projects are good and which are bad.
- $NPV > 0 \Rightarrow$ take the project.
- Say we've decided to take a particular project. How should we finance it?
 - What capital should we **raise**?
- Capital structure decisions relate to the optimal mix of debt/equity in financing the project.

Motivating questions

- Is there some optimal level of leverage such that we maximise the value of the firm?
- Can firms create value by borrowing money and paying a dividend/undertaking a share repurchase?
- Can value be created by issuing equity and repurchasing debt?
- Can value be created by paying dividends out of earnings?

Traditional view of capital structure

- The view of the 1950s and earlier.
- Equity is **expensive**.
- It's desirable to substitute for cheaper sources of funding such as debt.
- At some point, there are costs associated with excessive leverage, (e.g. high volatility of equity payouts, probability of bankruptcy).
- So there must be some **optimal level of leverage**.



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THE COST OF CAPITAL, CORPORATION FINANCE AND THE THEORY OF INVESTMENT

By FRANCO MODIGLIANI AND MERTON H. MILLER*

- But then this paper came along...
- A truly revolutionary paper by the two fathers of modern corporate finance.
- Both authors won Nobel Prizes in Economics.

Modigliani & Miller proposition 1 (1958)

- *“The total value of the securities issued by a firm is independent of the firms choice of capital structure. The firms value is determined by its real assets and growth opportunities, not by the types of securities it issues”*



- The firm's value is the size of a pizza: unless we get new real investment opportunities, the size won't change.
 - Capital structure changes the composition of flavours only, (e.g. half pepperoni/half mac and cheese v.s. 1/4 pepperoni and 3/4 mac and cheese).

Modigliani & Miller assumptions

- Does this mean that corporate finance is a redundant discipline?
- No! This proposition only holds under certain conditions:
 - (1) Perfect and complete capital markets.
 - (2) No taxes.
 - (3) Bankruptcy is not costly.
 - (4) Capital structure doesn't affect investment decisions and cash flows.
 - (5) Symmetric information between insiders and outsiders.

Modigliani & Miller proposition 1 proof (1)

- Idea behind the proof: if two portfolios have exactly the **same cash flows** then they must have the **same value**.
- Consider two firms — **Firm Levered** and **Firm Unlevered**.
 - Firm Unlevered: 100% equity financed.
 - Firm Levered: financed with debt, (due next year), and equity.
- Assume that both firms pay all their cash flows out as either dividends to equity or to debtholders, (if they have any).
- Say that each firm realises a cash flow of $\$X$.
- Assume that Firm Levered's debt has face value of $\$F < \X .

Modigliani & Miller proposition 1 proof (2)

- Consider **Portfolio A**:
 - Buy 1% of Firm Unlevered's equity (1% of V_U).
 - Cash flow of $\$0.01X$.

- Consider **Portfolio A'**:
 - Buy 1% of Firm Levered's equity (1% of E_L) and 1% of Firm Levered's debt (1% of D_L).
 - Cash flow of $\$0.01(X-F) + \$0.01F = \$0.01X$.

- Define the value of Firm Levered by V_L .

- It must then hold that the value of the two portfolios are equal:

$$\begin{aligned}0.01 V_U &= 0.01 E_L + 0.01 D_L \\ \Rightarrow V_U &= E_L + D_L \\ \Rightarrow V_U &= V_L\end{aligned}$$

which says that the two firms have the same value.

Modigliani & Miller proposition 1 proof (3)

- Consider **Portfolio B**:
 - Buy 1% of Firm Levered's equity (1% of E_L).
 - Cash flow of $\$0.01(X-F)$.

- Consider **Portfolio B'**:
 - Buy 1% of Firm Unlevered's equity (1% of V_U) and borrow/short sell 1% of Firm Levered's debt (1% of D_L).
 - Cash flow of $\$0.01(X-F)$.
 - I.e. receive $0.01X$ from Unlevered but need to pay back $0.01F$ from Levered's debt.

- Again both portfolios deliver the same cash flow, meaning that

$$\begin{aligned}E_L &= V_U - D_L \\ \Rightarrow E_L + D_L &= V_U \\ \Rightarrow V_L &= V_U.\end{aligned}$$

Where did we use the assumptions?

- (1) Perfect and complete capital markets: no transaction costs such that investors can form any portfolios they want.
- (2) No taxes: no distortions to the cash flow payouts to stakeholders.
- (3) Bankruptcy is not costly: in the event of bankruptcy, (if $F > X$ possible), then the debtholders assume control of the firm.
 - Didn't consider in these particular examples.
 - Exercise!
- (4) Capital structure doesn't affect investment decisions and cash flows: both of the firms have the same cash flows next period.
- (5) Symmetric information between insiders and outsiders: investors knew payouts next period.
 - Argument relies on **no arbitrage**: that the portfolios with same cash flows tomorrow must have the same value today.

Modigliani & Miller proposition 2

- The return on equity can be found by “re-levering”

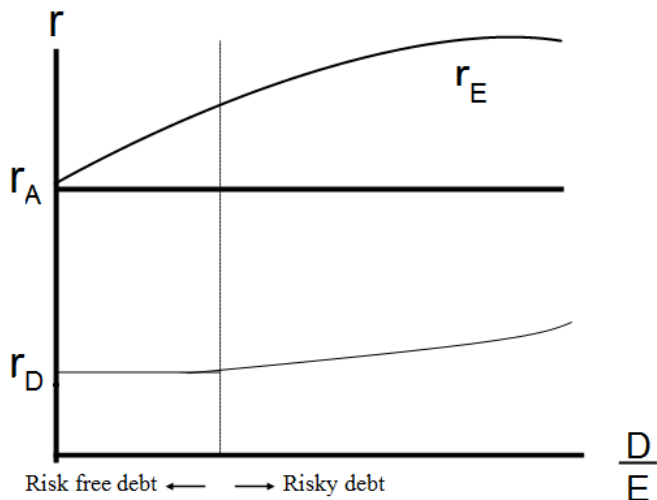
$$r_E = r_A + \frac{D}{E}(r_A - r_D)$$

- What does this look like? Recall from earlier

$$\beta_E = \beta_A + \frac{D}{E}(\beta_A - \beta_D)$$

- Return to equity is the sum of business risk and financial risk.

Modigiani & Miller proposition 2 pictorially



Modigliani & Miller and the balance sheet identities

- Asset additivity holds (for $j \in \{1, 2, \dots, n\}$ projects)

$$A = \sum_{j=1}^n A^j.$$

- Asset value also equals the sum of NPVs

$$A = \sum_{j=1}^n NPV^j.$$

- The balance sheet identity also holds

$$A = D + E \tag{1}$$

- MM propositions say that if we're not changing the left side of (1) then arbitrary changes to its right side won't affect firm value!

Example I: share repurchase

- EGI Inc. is financed solely by common stock. EGI has 5m shares outstanding with a market price of \$20 per share. EGI now announces that it intends to issue \$10m of debt, and use the proceeds to repurchase shares. Assume that all of the Modigliani & Miller assumptions hold.
 - (1) What is the market value ($E + D$) after this change in capital structure?
 - (2) How is the market price of the stock affected by the announcement?
 - (3) How many shares can MM Inc. repurchase with the \$10m million of new debt that we issue?
 - (4) Who (if anyone) gains or loses?

Example I solution (1)

(1) Market value will be unchanged due to MM theorem. Market value previously was equal to market capitalisation — $\$20 * 5m = \$100m$. Now notice though that after the repurchase, $\$10m$ of that total will comprise debt.

(2/3) We can set up two equations in two unknowns.

- Denote new price per share as P_{new} .
- Denote the number of shares repurchased as R .
- We know the following equations hold

$$P_{new} = \frac{\$100 - \$10}{5 - R} \quad (2)$$

$$P_{new} * R = \$10. \quad (3)$$

which give the equation for price per share after the repurchase and that the total amount we raise through debt will equal the value of the share repurchase. Notice that I've dropped the millions units for ease of exposition.

Example I solution (2)

- We can re-arrange equation (3) for P_{new} to obtain

$$P_{new} = \frac{\$10}{R} \quad (4)$$

- We can then equate equations (2) and (4) to obtain

$$\begin{aligned} \frac{100 - 10}{5 - R} &= \frac{10}{R} \\ \Rightarrow \frac{90}{5 - R} &= \frac{10}{R} \\ \Rightarrow 90R &= 50 - 10R \\ \Rightarrow 100R &= 50 \\ \Rightarrow R &= 0.5, \end{aligned}$$

which says we've repurchased half a million shares. We can then plug this into the equation (4) to get $P_{new} = \frac{10}{0.5} = \20 per share.

- Share price is unchanged??!? Is this wrong??

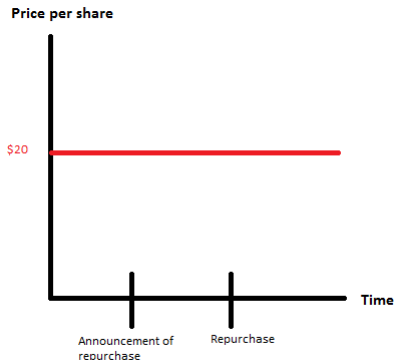
Example I solution (3)

(4) Nobody wins or loses!

- The debtholders hand-over \$10m and get a \$10m stake in the firm — indifferent.
- The equityholders who sell their shares receive \$10m collectively.
- This was the fair value of the shares originally, (recall 0.5m shares at \$20 per share).
- The shareholders who don't sell their shares are also indifferent — the price per share is the same.
- **This only happens because the MM assumptions all hold!**

Timing and stock price changes

- Investors are **forward-looking**.
- There is **perfect information**.
- They know the price will be unaffected at the time of the repurchase.
- So the price will be unaffected **at announcement**.



Example II: EPS ratio

- EGII Enterprises generates cash flows of \$5m per year in perpetuity starting next period onwards.
- Market value of its debt is \$10m, which is rolled-over perpetually at $r_D = 4\%$, (riskless rate).
- 4m shares are on issue at a price of \$10 per share.
 - (a) Solve for r_A .
 - (b) Solve for the earnings per share (EPS).
 - (c) Solve for the price to earnings ratio PE.
 - (d) Solve for r_E .

Example II: EPS ratio solutions (1)

(a) Notice firstly that the value of the firm is given by

$$\begin{aligned}V &= D + E \\ &= 10 + 40 \\ &= 50.\end{aligned}$$

Using the perpetuity formula, see that then that

$$\begin{aligned}V &= \frac{5}{r_A} \\ \Rightarrow 50 &= \frac{5}{r_A} \\ \Rightarrow r_A &= \frac{5}{50} \\ &= 10\%.\end{aligned}$$

where the second line comes from equating the left-side with the value we found above ($V = 50$).

Example II: EPS ratio solutions (2)

(b) Earnings per share can be found by

$$\begin{aligned}EPS &= \frac{\text{Cash flows} - \text{interest}}{\text{Number of shares}} \\ &= \frac{5 - 0.04(10)}{4} \\ &= \$1.15 \text{ per share.}\end{aligned}$$

(c) Price-earnings ratio then given by

$$\begin{aligned}PE &= \frac{\text{Share price}}{\text{EPS}} \\ &= \frac{10}{1.15} \\ &= 8.70.\end{aligned}$$

Example II: EPS ratio solutions (3)

(d) Recall the MM proposition 2 equation given by

$$\begin{aligned}r_E &= r_A + \frac{D}{E}(r_A - r_D) \\ &= 10\% + \frac{1}{4}(10\% - 4\%) \\ &= 11.5\%.\end{aligned}$$

Example II: EPS ratio [part 2]

- Suppose now that the CEO decides to try to increase the PE ratio to “increase shareholder value” .
- The firm will achieve this by issuing new shares any buying-back all outstanding debt.
 - (i) How many shares do we need to issue, and what happens to the share price?
 - (ii) What happens to the PE ratio?
 - (iii) What happens to r_E ?

Example II: EPS ratio [part 2] solutions (1)

- (i) Again let's go for the two equations and two unknowns approach. Let's let S be the number of new shares issued. Then

$$P_{new} = \frac{50}{4 + S}$$
$$P_{new}S = 10$$

where the first equation uses the fact that all the debt will be repurchased — the new equity value is the firm value. You can then solve these two equations to get that $S = 1m$ and $P_{new} = \$10$.

- (ii) The EPS ratio is now $EPS = \frac{5}{5} = \$1$ per share.
- (iii) The PE ratio is now $PE = \frac{10}{1} = 10$.
- (iv) The return to equity is now $r_E = 10\% + 0 = 10\%$.

Example II: EPS ratio [part 2] solutions (2)

- What are the takeaways from this part of the problem?
- Indeed the CEO was able to increase the PE ratio.
- While this may fool some investors, the firm hasn't actually created any value.



Example III: generalised proposition 2 formula

- Say a firm is financed with equity and two types of **different debt** — with values denoted by E , D^1 and D^2 respectively.
- The firm's returns to assets, equity, the first and second types of debt are r_A , r_E , r_D^1 and r_D^2 respectively.
- Derive an expression for r_E in terms of the other returns!
- **Hint:** notice that the balance sheet identity for this particular firm will be $V = E + D^1 + D^2$.

Example III: generalised proposition 2 formula solution

- We can start with the expression for r_A , which says that r_A is a weighted average of the returns to the other stakeholders of the firm

$$\begin{aligned}r_A &= \frac{E}{E + D^1 + D^2} r_E + \frac{D^1}{E + D^1 + D^2} r_D^1 + \frac{D^2}{E + D^1 + D^2} r_D^2 \\ \Rightarrow \frac{E}{E + D^1 + D^2} r_E &= r_A - \frac{D^1}{E + D^1 + D^2} r_D^1 - \frac{D^2}{E + D^1 + D^2} r_D^2 \\ \Rightarrow r_E &= \frac{E + D^1 + D^2}{E} r_A - \frac{D^1}{E} r_D^1 - \frac{D^2}{E} r_D^2 \\ &= \frac{E}{E} r_A + \frac{D^1}{E} r_A + \frac{D^2}{E} r_A - \frac{D^1}{E} r_D^1 - \frac{D^2}{E} r_D^2 \\ &= r_A + \frac{D^1}{E} (r_A - r_D^1) + \frac{D^2}{E} (r_A - r_D^2),\end{aligned}$$

which is a generalisation of the MM proposition 2 formula!

Is capital structure really irrelevant?

- Obviously not.
- It's just a benchmark.
- The next questions people started asking were — “what happens as we start relaxing these assumptions?”
- That's where we go from here for the rest of the **theoretical** part of the course.

Takeaways

- Capital structure is concerned with how we finance projects.
- Under certain conditions, capital structure of a firm is irrelevant.
- Capital structure only matters **when the MM assumptions fail**.