# FIN 325 Corporate Finance <br> L6 (Theory): Tax Benefits of Debt under APV Method 

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## Motivation

- In L6 we saw that firm value was independent of capital structure under certain conditions.
- The traditional assumption to relax first is that there are no taxes.
- Perhaps the most obvious violation of the MM assumptions!


## Corporate tax rates across the OECD

Corporate Income Tax Rates in the OECD


## Tax deductibility of interest (1)

- Interest payments are tax deductible in the United States.
- Paid out of before tax earnings.
- Holding all else equal, a higher level of interest payments reduces the firm's taxable income.
- Debt financing has an additional benefit over equity financing along this channel.


## Tax deductibility of interest (2)

- Consider two firms - Firm Unlevered and Firm Levered.
- Firm Unlevered is $100 \%$ equity while Firm Levered borrowed $\$ 1,000$ worth of debt at $8 \%$ interest.
- Assume that capital expenditures, depreciation and the change in net working capital are zero.

| EBIT | Firm Unlevered | Firm Levered |
| :---: | :---: | :---: |
| Interest | 500 | 500 |
| Pretax income | 0 | 80 |
| Tax (35\%) | 500 | 420 |
| Net income to shareholders | 175 | 147 |
| Total income to D and E | $325+0=325$ | $273+80=353$ |
| Tax shield from debt | 0 | 28 |

- Total cash flow from Firm Levered is 28 higher - known as the debt tax shield (DTS).


## Tax deductibility of interest (3)

- Firm Unlevered is $100 \%$ equity while Firm Levered borrowed \$D worth of debt at $r_{D} \%$ interest.

| EBIT | Firm Unlevered | Firm Levered |
| :---: | :---: | :---: |
| Interest | $C$ | $C$ |
| Pretax income | 0 | $r_{D} D$ |
| Tax (35\%) | $C$ | $C-r_{D} D$ |
| Net income to shareholders | $\left(1-\tau^{C}\right) C$ | $\left(1-\tau^{C}\right)\left(C-r_{D} D\right)$ |
| Total income to D and E | $\left(1-\tau^{C}\right) C$ | $\left(1-\tau^{C}\right) C+\tau^{C} r_{D} D$ |
| Tax shield from debt | 0 | $\tau^{C} r_{D} D$ |

- Value from having debt of $D$ at interest rate of $r_{D}$ is $\tau^{C} r_{D} D$.


## Valuation of tax shields

- If a firm takes-out some debt worth $D$ at an interest rate of $r_{D}$ for $T$ periods

$$
\begin{equation*}
P V(D T S)=\sum_{t=1}^{T} \frac{\tau^{C} r_{D} D}{\left(1+r^{*}\right)^{t}} . \tag{1}
\end{equation*}
$$

- If the debt is assumed to be held in perpetuity, then

$$
\begin{equation*}
P V(D T S)=\frac{\tau^{C} r_{D} D}{r^{*}} \tag{2}
\end{equation*}
$$

- What discount rate should we use for $r^{*}$ in (1) and (2)?


## Discount rate for tax shields

- What discount rate should we use for $r^{*}$ in the tax shields formula?
- The rate should reflect the riskiness of the tax shield payments.
(1) $r_{D}$ : reflects the risk of the debt that generates the tax shields.
(2) $r_{A}$ : reflects the risk of the corporate profits, which we need to generate tax shields.
(3) $r_{E}$ : gives a conservative estimate of the risk.
- When we set $r^{*}=r_{D}$ then the formula for (2) simplifies

$$
P V(D T S)=\tau^{C} D
$$

## Adjusted present value method (APV) (1)

- A method for evaluating firms and projects.
- Say that a project is to be financed using debt and equity.
- APV method looks at a counterfactual where the project is $100 \%$ equity-financed then adds-in the tax benefits of debt.
- Explicitly separates the project's cash flows that are associated with real operations and financing.


## Adjusted present value method (APV) (2)

- Refer back to slide (6) with Firm Unlevered and Firm Levered.
- Let's assume that their cash flows are perpetual.
- I.e. each firm has project cash flows of $C$ each period while Firm Levered has debt of $D$ maintained in perpetuity.
- Denote the value of Firm Unlevered and Firm Levered by $V_{U}$ and $V_{L}$ respectively.

$$
\begin{aligned}
& V_{U}=P V\left[\left(1-\tau^{C}\right) C\right] \\
& V_{L}=P V\left[\left(1-\tau^{C}\right) C+\tau^{c} r_{D} D\right]
\end{aligned}
$$

which can be combined to get

$$
\begin{equation*}
V_{L}=V_{U}+P V(D T S) \tag{3}
\end{equation*}
$$

which is known as the APV formula.

- We'll make some more adjustments to equation (3) in future lectures.


## Some notes on APV

- When calculating the APV of a new project, always use the incremental debt.
- $\tau^{C}$ is the marginal tax rate; not the average.
- Need to look at the tax that you'll be charged on marginal debt.
- When we have positive taxes, we can create value by purely changing firm's capital structure, (see example later).


## Personal taxes (1)

- Investors in the firm need to pay additional taxes, (e.g. capital gains and dividends).
- The cash flow to individual investors net of all taxes is what's important.

| Time Period | C-Corporation |  | Individual |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Ordinary Income $\left(\tau_{c}\right)$ | Capital Gains ( $\tau_{\mathrm{c}, \mathrm{cg}}$ ) | Ordinary Income $\left(\tau_{p}\right)$ | Dividends ( $\tau_{\text {p,div }}$ ) | Capital Gains ( $\tau_{\mathrm{p}, \mathrm{cg}}$ ) |
| Pre-1981 | 46.0\% | 28.0\% | 70.0\% | 70.0\% | 28.0\% |
| 1982-1986 | 46.0\% | 20.0\% | 50.0\% | 50.0\% | 20.0\% |
| 1987 | 40.0\% | 28.0\% | 39.0\% | 39.0\% | 28.0\% |
| 1988-1990 | 34.0\% | 34.0\% | 28.0\% | 28.0\% | 28.0\% |
| 1991-1992 | 34.0\% | 34.0\% | 31.0\% | 31.0\% | 28.0\% |
| 1993-1996 | 35.0\% | 35.0\% | 39.6\% | 39.6\% | 28.0\% |
| 1997-2000 | 35.0\% | 35.0\% | 39.6\% | 39.6\% | 20.0\% |
| 2001-2002 | 35.0\% | 35.0\% | 38.6\% | 38.6\% | 20.0\% |
| 2003- | 35.0\% | 35.0\% | 35.0\% | 15.0\% | 15.0\% |

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## Personal taxes (2)

- Denote $\tau^{i}$ the personal tax rate paid on interest/ordinary income.
- Denote $\tau^{e}$ the personal tax rate on dividends.

|  | Flow to debtholders | Flow to equityholders |
| :---: | :---: | :---: |
| Amount to distribute | 1 | 1 |
| Corporate taxes | 0 | $\tau^{c}$ |
| Income after corporate tax | 1 | $1-\tau^{c}$ |
| Personal taxes | $\tau^{i}$ | $\left(1-\tau^{c}\right)\left(1-\tau^{e}\right)$ |
| Investor flow after tax | $\left(1-\tau^{i}\right)$ | $\left(1-\tau^{c}\right)\left(1-\tau^{e}\right)$ |

- See that there is no corporate tax liability for interest paid to debtholders.
- Tax benefit to debt if $\left(1-\tau^{i}\right)>\left(1-\tau^{C}\right)\left(1-\tau^{e}\right)$.
- Has been the case historically.


## Personal taxes (3)

- When debt is held in perpetuity then

$$
\begin{aligned}
P V(D T S) & =D \tau^{*} \\
\tau^{*} & =\frac{\left(1-\tau^{i}\right)-\left(1-\tau^{c}\right)\left(1-\tau^{e}\right)}{1-\tau^{i}},
\end{aligned}
$$

which is expressed as a percentage. Derivation from Miller (1977).

- When $\tau^{i}$ is high, the tax advantage of debt is smaller.
- When $\tau^{e}=\tau^{i}$ then $P V(D T S)=\tau^{C} D$ as before.


## Personal taxes (4)

- How does $\tau^{*}$ vary with $\tau^{i}$ ?
- Fix $\tau^{C}=0.35$ and $\tau^{e}=0.15$ and vary $\tau^{i}$ from 0 to 0.5 (below).



## What about negative earnings before tax (EBT)?

- EBT defined as EBIT less interest expenses.
- If this measure is negative, then things get complicated...
- Need to take account of carryforwards and carrybacks for the operating losses.
- Do we deduct the losses against previous or future losses?
- We won't worry about this case in the course.


## Example 1: leveraged recapitalisation

- Firm Pure has 1 b shares outstanding, which are trading at $\$ 640$ per share.
- The firm is currently $100 \%$ equity.
- Assume that $\tau^{C}=0.35$ and $r_{D}=0.03$ while all personal tax rates are zero.
- Say that the firm issues $\$ 100$ b in perpetual debt and uses the proceeds to repurchase shares.
(a) What happens to the share price? What is the value of the debt tax shield?
(b) What is the firm's value after the recap?
(c) How many shares are repurchased and at what price?
(d) Who gains and loses from the recap?
(e) What happens to the price at announcement?


## Example 1 solution (1)

- The firm was originally $100 \%$ equity so $V_{U}=\$ 640 \times 1 b=\$ 640 b$.
- With the new debt, we can use APV to get

$$
\begin{aligned}
V_{L} & =V_{U}+P V(D T S) \\
& =\$ 640 b+\$ 100(0.35) b \\
& =\$ 675 b
\end{aligned}
$$

where I just assumed that the debt was perpetual and $r_{D}$ was the appropriate discount rate.

- Use the two equations and two unknowns approach to find the new share price and the number of shares repurchased.

$$
\begin{aligned}
S \times P_{\text {new }} & =\$ 100 b \\
P_{\text {new }} & =\frac{\$ 675 b-\$ 100 b}{1 b-S}
\end{aligned}
$$

which can be solved to get $P_{\text {new }}=\$ 675$ and $S=\frac{100}{675} b$.

## Example 1 solution (2)



- Price response at time of announcement due to forward-looking investors.
- Assumes that the announcement comes as a surprise to the investors!


## Example 1 [part 2]

- Now assume that there are personal tax rates of $\tau^{e}=0.15$ and $\tau^{i}=0.35$. Repeat the previous exercise!
- Give an intuition for the different price that the shares rise to on announcement relative to part 1 .


## Example 1 [part 2] solution (1)

- The effective tax benefit of debt is given by

$$
\begin{aligned}
\tau^{*} & =1-\frac{\left(1-\tau^{C}\right)\left(1-\tau^{e}\right)}{1-\tau^{i}} \\
& =0.15
\end{aligned}
$$

- Then the present value of the DTS is $P V(D T S)=\$ 15 b$.
- $V_{L}=\$ 640 b+\$ 15 b=\$ 655 b$.

$$
\begin{aligned}
S P_{\text {new }} & =\$ 100 b \\
P_{\text {new }} & =\frac{655-100}{1-S}
\end{aligned}
$$

which can be solved for $S=\frac{100}{655}$ and $P_{\text {new }}=\$ 655$.

- The price rise is now smaller given that there is less of a tax benefit of debt due to $\tau^{i}>0$.


## Takeaways

- Introduce taxes into our capital structure model: debt has an advantage over equity.
- APV method says that $V_{L}=V_{U}+P V(D T S)$.
- But then the firm should logically max-out on debt relative to equity! Take out 99.9999... \% debt!
- Why don't firms do this in reality?.... Hold your breath until a couple of lectures time!



[^0]:    ${ }^{1}$ Departments of Economics and Finance, UW-Madison.

[^1]:    Source: Scholes et al. (2005), Taxes and Business Strategy ( $3^{\text {rd }} \mathrm{ed}$ ), Table 1.1, p. 12

