Lecture 9: New Keynesian Model Part II
Price Stickiness

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Roadmap

Motivation

- Last time we studied monopolistic competition in a static framework.
- If we were to extend this basic setup into a dynamic setting, (without including any other frictions), firms would adjust their prices each period.
- Now let's explore what happens when firms can no longer perfectly adjust their prices.

This Lecture

- We'll study two standard ways of capturing price rigidities.
  (1) Calvo price stickiness,
  (2) Rotemberg price stickiness.

Why are we Doing This?

- The old Keynesian paradigm pushes that monetary policy has an impact due to price rigidities.
- We want to formalise this idea.
- Last lecture: firms choose their own prices.
- Now what happens when we combine this with nominal frictions?
- Next lecture: how does this sticky price-setting spill-over to impact the macroeconomy?

Roadmap

Setup

- Reduced-form way of capturing price rigidities.
- Each period, a given firm has probability of θ that they will have the same price as last period.
- Complementary probability $1-\theta$ that they will be able to update their price.
- Green light for adjustment known colloquially as "receiving a visit from the Calvo fairy".
- Calvo (1983), "Staggered Prices in a Utility-Maximising Framework", *Journal of Monetary Economics*.

Law of Motion for the Price Level

- Firms in the model who update will all choose the same optimal price. Why? They're all effectively the same.
- Denote the optimal price by P_t^*.
- Recall the aggregate price index from the last lecture. Denote the set of firms, who keep the same price as last period, as $S(t) \subset [0,1]$ (with $S(t)'$ the remainder).

$$\begin{aligned} P_t &= \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \\ &= \left(\int_{S(t)} P_t(j)^{1-\epsilon} dj + \int_{S(t)'} P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \\ &= \left[\theta (P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \end{aligned}$$

- What is the optimal reset price, P_t^*?

Firm Objective

- Recall from last lecture that the firm objective was to simply maximise static profits.
- In a dynamic context without price rigidities, the objective is the same. Why?
- With price stickiness, we need to form some expectation over future profits though.
- In this dynamic context, assume productivity follows the process

$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_{a,t}, \;\; \epsilon_{a,t} \sim N(0, \sigma_a^2)$$

Firm Objective

- The firm aims to maximise the discounted value of expected future profits.
- Need to take account of how the choice of optimal price today will impact profits in the future, conditional on being getting a sequence of red lights.
- What are the essential ingredients to calculating this object?
- Think of NPV analysis used in corporate finance/business classes to find the market value of a sequence of cash flows. We need to know:
  - The cash flow values for each period,
  - The appropriate discount factor.

Firm Objective

- In this case, the cash flows each period are the profits of the firm conditional on having the optimal price chosen at t.
- The discount factor is supposed to represent the opportunity cost of funds used in the project.
  - The relevant agents to consider are the owners of the equity in the firm: the households here in this model.
  - Recall the consumption Euler equation for the households in the MIU model

$$q_t = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} (1) \right]$$

    where 1 was the nominal payoff of a bond and q_t was its price.
- The object $\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$ is referred to as the one period ahead nominal stochastic discount factor for the household.

Firm Objective

- The value of the profits given the choice of P_t^* is found by discounting the k period ahead profits of the firm using the k period stochastic discount factor.

- Expected discounted profits

$$\Gamma_t(j) = \mathcal{Q}_{t\to t} V_{t,t}(j) + \theta \mathbb{E}_t[\mathcal{Q}_{t\to t+1} V_{t,t+1}(j)] + \theta^2 \mathbb{E}_t[\mathcal{Q}_{t\to t+2} V_{t,t+2}(j)] + \ldots$$
$$= \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t\to t+k} V_{t,t+k}(j) \right\}$$

where $V_{t,t+k}(j)$ is profit at $t+k$ with price chosen at t and $\mathcal{Q}_{t\to t+k}$ is the k period ahead stochastic discount factor

$$\mathcal{Q}_{t\to t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$$

k Period Ahead Profits

- What is the expression for $V_{t,t+k}(j)$?
- Firm faces demand curve at period $t+k$ given the optimal price set at t

$$Y_{t,t+k}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

where $Y_{t,t+k}(j)$ denotes the demand for the firm's variety at $t+k$ given the price set at t and Y_{t+k} is aggregate output at $t+k$.

k Period Ahead Profits

- Can then write $V_{t,t+k}(j)$ as

$$V_{t,t+k}(j) = P_t^* Y_{t,t+k}(j) - TC_{t+k}(Y_{t,t+k}(j))$$

where $TC_{t+k}(Y_{t,t+k}(j))$ is the total cost at $t+k$.

- This is the total cost given factor prices at $t+k$ for producing the amount $Y_{t,t+k}(j)$ — the firm's level of demand given its time t reset.

Optimal Price

- FOC

$$\frac{\partial \Gamma_t(j)}{\partial P_t^*} = 0 \Rightarrow \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t \to t+k} \frac{\partial V_{t,t+k}(j)}{\partial P_t^*} \right\} = 0$$

where

$$\frac{\partial V_{t,t+k}(j)}{\partial P_t^*} = Y_{t,t+k}(j) \left[(1-\epsilon) + \epsilon \frac{1}{P_t^*} TC'_{t+k}(Y_{t,t+k}(j)) \right].$$

Since

$$\begin{aligned} \frac{\partial Y_{t,t+k}(j)}{P_t^*} &= (-\epsilon) \frac{1}{P_{t+k}} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon-1} Y_{t+k} \\ &= -\frac{\epsilon}{P_t^*} Y_{t,t+k}(j) \end{aligned}$$

Optimal Price

- Why is there no derivative of the stochastic discount factor here?

Where to From Here?

- This FOC forms the basis for what's known as the new Keynesian Phillips curve.
- It's traditionally linearised: we'll do this next lecture.

Roadmap

Setup

- This model assumes that firms can always opt to change their price, but doing so involves paying a quadratic adjustment cost of the form

$$AC_t(j) = \frac{\lambda}{2}\left(\frac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*} - 1\right)^2 Y_t$$

  where $\lambda > 0$ captures the degree of price stickiness and $\widetilde{P}_t^*$ denotes the optimal reset price.

- Rotemburg (1982), "Sticky Prices in the United States", *Journal of Political Economy*.

Firm Objective

- What are the cash flows we need to think about discounting in this problem?
- The choice of a new price today affects our profits today.
- But it can also affect our profits directly tomorrow. Why?
- Because in $t+1$, the re-set price $\widetilde{P}_t^*$ will feature in the adjustment cost function

$$AC_{t+1}(j) = \frac{\lambda}{2}\left(\frac{\widetilde{P}_{t+1}^*}{\widetilde{P}_t^*} - 1\right)^2 Y_{t+1}$$

where I've just updated the time subscripts from earlier by one.

Firm Objective

- Any other future periods where today's price choice will have a direct effect?
- What about time $t+2$?
- Nope. The price chosen at $t+1$ will affect that (as in the previous slide).
- So price chosen at time t won't impact profits at $t+2$ directly.
- All these future derivatives will drop out from the FOC.

Firm Objective

- Again, firm seeks to maximise the discounted expected value of future profits for its shareholders

$$\widetilde{\Gamma}_t(j) = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{Q}_{t \to t+k} \widetilde{V}_{t+k}(j) \right\}$$

where

$$\widetilde{V}_{t+k}(j) = \widetilde{P}_t^* Y_{t+k}(j) - TC_{t+k}(Y_{t+k}(j)) - P_{t+k} AC_{t+k}(j)$$

and

$$Y_{t+k}(j) = \left(\frac{\widetilde{P}_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$$

- Why am I multiplying the adjustment cost by the price index?

Optimal Price

- FOC

$$\frac{\partial \widetilde{\Gamma}_t(j)}{\partial \widetilde{P}_t^*} = 0 \Rightarrow \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{Q}_{t \to t+k} \frac{\partial \widetilde{V}_{t+k}(j)}{\partial \widetilde{P}_t^*} \right\} = 0$$

where

$$\frac{\partial \widetilde{V}_t(j)}{\partial \widetilde{P}_t^*} = Y_t(j) + \widetilde{P}_t^* \frac{\partial Y_t(j)}{\partial \widetilde{P}_t^*} + TC_t'(Y_t(j)) \frac{\partial Y_t(j)}{\partial \widetilde{P}_t^*} - \lambda \left(\frac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*} - 1 \right) Y_t \frac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*}$$

$$\frac{\partial \widetilde{V}_{t+1}(j)}{\partial \widetilde{P}_t^*} = \lambda \frac{\widetilde{P}_{t+1}^*}{(\widetilde{P}_t^*)^2} \left(\frac{\widetilde{P}_{t+1}^*}{\widetilde{P}_t^*} - 1 \right) P_{t+1} Y_{t+1}$$

Optimal Price

- Putting it all together yields (exercise: check)

$$\left(\frac{(\widetilde{P}_t^*)}{P_t}\right)^{-\epsilon} Y_t - \frac{\widetilde{P}_t^*}{P_t}\epsilon\left(\frac{(\widetilde{P}_t^*)}{P_t}\right)^{-\epsilon-1} Y_t + TC_t'(Y_t(j))\epsilon\left(\frac{(\widetilde{P}_t^*)}{P_t}\right)^{-\epsilon-1} Y_t\frac{1}{P_t} -$$
$$\lambda\left(\frac{(\widetilde{P}_t^*)}{(\widetilde{P}_{t-1}^*)}-1\right)\frac{Y_tP_t}{(\widetilde{P}_{t-1}^*)} + \lambda\mathbb{E}_t\left[Q_{t\to t+1}\frac{(\widetilde{P}_{t+1}^*)}{(\widetilde{P}_t^*)^2}\left(\frac{(\widetilde{P}_{t+1}^*)}{(\widetilde{P}_t^*)}-1\right)Y_{t+1}P_{t+1}\right]$$
$$= 0$$

Roadmap

Cross-Sectional Price Dispersion

- Is there a cross-section of different prices across the varieties?
- Rotemburg: none. Why? Firms will just re-adjust each period. All the same, so they set the same price.
- Calvo: in general, there will be dispersion. E.g. consider initial price index P_0.

$$\Rightarrow P_1 = \left[\theta(P_0)^{1-\epsilon} + (1-\theta)(P_1^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P_2 = \left[\theta\{\theta(P_0)^{1-\epsilon} + (1-\theta)(P_1^*)^{1-\epsilon}\} + (1-\theta)(P_2^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$= \left[\theta^2(P_0)^{1-\epsilon} + \theta(1-\theta)(P_1^*)^{1-\epsilon} + (1-\theta)(P_2^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P_3 = \left[\theta^3(P_0)^{1-\epsilon} + \theta^2(1-\theta)(P_1^*)^{1-\epsilon} + \theta(1-\theta)(P_2^*)^{1-\epsilon} + (1-\theta)(P_3^*)^{1-\epsilon}\right]$$

and so on.

Induced Distortions

- Adjustment costs in Rotemburg are goods that come out of the resource constraint

$$
\begin{aligned}
Y_t &= C_t + \int_0^1 AC_t(j)dj \\
&= C_t + \int_0^1 \frac{\lambda}{2}\left(\frac{(\widetilde{P}_t^*)}{(\widetilde{P}_{t-1}^*)} - 1\right)^2 Y_t dj \\
&= C_t + Y_t \frac{\lambda}{2} \int_0^1 \left(\frac{P_t}{P_{t-1}} - 1\right)^2 dj \\
&= C_t + Y_t \frac{\lambda}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 \\
\Rightarrow Y_t &= \left\{1 - \frac{\lambda}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2\right\}^{-1} C_t
\end{aligned}
$$

i.e. there is an "inefficiency wedge" between output and consumption.

Induced Distortions

- Under the Calvo model, price dispersion creates distortions.
- Recall from the production function that

$$Y_t(j) = A_t N_t(j)^{1-\alpha}$$
$$\Rightarrow N_t(j) = \left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{1-\alpha}}$$

Which is labour demand for firm j. Aggregation gives

$$N_t = \int_0^1 N_t(j)dj = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{1-\alpha}} dj$$
$$= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj$$

where the last line comes from plugging-in j's demand function.

Induced Distortions

- Notice that if there is perfect price flexibility, then

$$\int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj = \int_0^1 \left(\frac{P_t}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj = 1$$

- With rigidities though, see that

$$N_t^{1-\alpha} = \left(\frac{Y_t}{A_t}\right) \left\{ \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj \right\}^{1-\alpha}$$

$$\Rightarrow Y_t = A_t N_t^{1-\alpha} \left\{ \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj \right\}^{\alpha-1}$$

where $\left\{ \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj \right\}^{\alpha-1} < 1$ [you need not show this].

- You can interpret the last equality as an aggregate production function.

Big Picture

- We want a model with price rigidity so we can think about non-neutral monetary policy.
- Ok, but what does price stickiness itself imply about welfare?
- It's a bad thing.
- It's a friction: firms are unable to update their prices freely, even if they wanted to.
- This can only hurt our economy relative to a benchmark without price rigidity.

Big Picture

- The setup of these two models allows us to think a bit about what the welfare cost of this friction is.
- In Calvo, the dispersion hurts welfare.
- In Rotemburg, the adjustment costs hurt welfare.

Roadmap

Takeaways

- We looked at two forms of modelling price stickiness.
- Both create distortions.
- Different sources of distortion though.