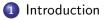
Lecture 9: New Keynesian Model Part II Price Stickiness

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Roadmap



2 Calvo Model

3 Rotemberg Model





Motivation

- Last time we studied monopolistic competition in a static framework.
- If we were to extend this basic setup into a dynamic setting, (without including any other frictions), firms would adjust their prices each period.
- Now let's explore what happens when firms can no longer perfectly adjust their prices.



This Lecture

- We'll study two standard ways of capturing price rigidities.
 - (1) Calvo price stickiness,
 - (2) Rotemberg price stickiness.

Why are we Doing This?

- The old Keynesian paradigm pushes that monetary policy has an impact due to price rigidities.
- We want to formalise this idea.
- Last lecture: firms choose their own prices.
- Now what happens when we combine this with nominal frictions?
- Next lecture: how does this sticky price-setting spill-over to impact the macroeconomy?

Roadmap





3 Rotemberg Model





Setup

- Reduced-form way of capturing price rigidities.
- Each period, a given firm has probability of θ that they will have the same price as last period.
- Complementary probability 1θ that they will be able to update their price.
- Green light for adjustment known colloquially as "receiving a visit from the Calvo fairy".
- Calvo (1983), "Staggered Prices in a Utility-Maximising Framework", Journal of Monetary Economics.

Law of Motion for the Price Level

- Firms in the model who update will all choose the same optimal price. Why? They're all effectively the same.
- Denote the optimal price by P_t^* .
- Recall the aggregate price index from the last lecture. Denote the set of firms, who keep the same price as last period, as S(t) ⊂ [0, 1] (with S(t)' the remainder).

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$
$$= \left(\int_{S(t)} P_t(j)^{1-\epsilon} dj + \int_{S(t)'} P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$
$$= \left[\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

• What is the optimal reset price, P_t^* ?

- Recall from last lecture that the firm objective was to simply maximise static profits.
- In a dynamic context without price rigidities, the objective is the same. Why?
- With price stickiness, we need to form some expectation over future profits though.
- In this dynamic context, assume productivity follows the process

$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_{a,t}, \ \epsilon_{a,t} \sim N(0, \sigma_a^2)$$

- The firm aims to maximise the discounted value of expected future profits.
- Need to take account of how the choice of optimal price today will impact profits in the future, conditional on being getting a sequence of red lights.
- What are the essential ingredients to calculating this object?
- Think of NPV analysis used in corporate finance/business classes to find the market value of a sequence of cash flows. We need to know:
 - The cash flow values for each period,
 - The appropriate discount factor.

- In this case, the cash flows each period are the profits of the firm conditional on having the optimal price chosen at *t*.
- The discount factor is supposed to represent the opportunity cost of funds used in the project.
 - The relevant agents to consider are the owners of the equity in the firm: the households here in this model.
 - Recall the consumption Euler equation for the households in the MIU model

$$q_t = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} (1) \right]$$

where 1 was the nominal payoff of a bond and q_t was its price.

• The object $\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}$ is referred to as the one period ahead nominal stochastic discount factor for the household.

- The value of the profits given the choice of P_t^* is found by discounting the k period ahead profits of the firm using the k period stochastic discount factor.
- Expected discounted profits

$$\Gamma_{t}(j) = \mathcal{Q}_{t \to t} V_{t,t}(j) + \theta \mathbb{E}_{t}[\mathcal{Q}_{t \to t+1} V_{t,t+1}(j)] + \theta^{2} \mathbb{E}_{t}[\mathcal{Q}_{t \to t+2} V_{t,t+2}(j)] + \dots$$
$$= \mathbb{E}_{t} \left\{ \sum_{k=0}^{\infty} \theta^{k} \mathcal{Q}_{t \to t+k} V_{t,t+k}(j) \right\}$$

where $V_{t,t+k}(j)$ is profit at t + k with price chosen at t and $Q_{t \to t+k}$ is the k period ahead stochastic discount factor

$$Q_{t \to t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$$

k Period Ahead Profits

- What is the expression for $V_{t,t+k}(j)$?
- Firm faces demand curve at period *t* + *k* given the optimal price set at *t*

$$Y_{t,t+k}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

where $Y_{t,t+k}(j)$ denotes the demand for the firm's variety at t + k given the price set at t and Y_{t+k} is aggregate output at t + k.

k Period Ahead Profits

• Can then write $V_{t,t+k}(j)$ as

$$V_{t,t+k}(j) = P_t^* Y_{t,t+k}(j) - TC_{t+k}(Y_{t,t+k}(j))$$

where $TC_{t+k}(Y_{t,t+k}(j))$ is the total cost at t + k.

 This is the total cost given factor prices at t + k for producing the amount Y_{t,t+k}(j) — the firm's level of demand given its time t reset.

Optimal Price

• FOC

$$\frac{\partial \Gamma_t(j)}{\partial P_t^*} = 0 \Rightarrow \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t \to t+k} \frac{\partial V_{t,t+k}(j)}{\partial P_t^*} \right\} = 0$$

where

$$\frac{\partial V_{t,t+k}(j)}{\partial P_t^*} = Y_{t,t+k}(j) \left[(1-\epsilon) + \epsilon \frac{1}{P_t^*} TC'_{t+k}(Y_{t,t+k}(j)) \right].$$

Since

$$\frac{\partial Y_{t,t+k}(j)}{P_t^*} = (-\epsilon) \frac{1}{P_{t+k}} \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon-1} Y_{t+k}$$
$$= -\frac{\epsilon}{P_t^*} Y_{t,t+k}(j)$$

Optimal Price

• Why is there no derivative of the stochastic discount factor here?

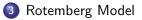
Where to From Here?

- This FOC forms the basis for what's known as the new Keynesian Phillips curve.
- It's traditionally linearised: we'll do this next lecture.

Roadmap











Setup

• This model assumes that firms can always opt to change their price, but doing so involves paying a quadratic adjustment cost of the form

$$\mathcal{AC}_t(j) = rac{\lambda}{2} \left(rac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*} - 1
ight)^2 Y_t$$

where $\lambda > 0$ captures the degree of price stickiness and \widetilde{P}_t^* denotes the optimal reset price.

• Rotemburg (1982), "Sticky Prices in the United States", *Journal of Political Economy*.

- What are the cash flows we need to think about discounting in this problem?
- The choice of a new price today affects our profits today.
- But it can also affect our profits directly tomorrow. Why?
- Because in t+1, the re-set price \widetilde{P}_t^* will feature in the adjustment cost function

$$\mathcal{AC}_{t+1}(j) = rac{\lambda}{2} \left(rac{\widetilde{P}^*_{t+1}}{\widetilde{P}^*_t} - 1
ight)^2 Y_{t+1}$$

where I've just updated the time subscripts from earlier by one.

- Any other future periods where today's price choice will have a direct effect?
- What about time *t* + 2?
- Nope. The price chosen at t + 1 will affect that (as in the previous slide).
- So price chosen at time t won't impact profits at t + 2 directly.
- All these future derivatives will drop out from the FOC.

• Again, firm seeks to maximise the discounted expected value of future profits for its shareholders

$$\widetilde{\Gamma}_t(j) = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{Q}_{t \to t+k} \widetilde{V}_{t+k}(j) \right\}$$

where

$$\widetilde{V}_{t+k}(j) = \widetilde{P}_t^* Y_{t+k}(j) - TC_{t+k}(Y_{t+k}(j)) - P_{t+k}AC_{t+k}(j)$$

and

$$Y_{t+k}(j) = \left(\frac{\widetilde{P}_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

• Why am I multiplying the adjustment cost by the price index?

Optimal Price

• FOC

$$\frac{\partial \widetilde{\Gamma}_t(j)}{\partial \widetilde{P}_t^*} = 0 \Rightarrow \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{Q}_{t \to t+k} \frac{\partial \widetilde{V}_{t+k}(j)}{\partial \widetilde{P}_t^*} \right\} = 0$$

where

$$\frac{\partial \widetilde{V}_{t}(j)}{\partial \widetilde{P}_{t}^{*}} = Y_{t}(j) + \widetilde{P}_{t}^{*} \frac{\partial Y_{t}(j)}{\partial \widetilde{P}_{t}^{*}} + TC_{t}'(Y_{t}(j)) \frac{\partial Y_{t}(j)}{\partial \widetilde{P}_{t}^{*}} - \lambda \left(\frac{\widetilde{P}_{t}^{*}}{\widetilde{P}_{t-1}^{*}} - 1\right) Y_{t} \frac{\widetilde{P}_{t}^{*}}{\widetilde{P}_{t-1}^{*}} \\ \frac{\partial \widetilde{V}_{t+1}(j)}{\partial \widetilde{P}_{t}^{*}} = \lambda \frac{\widetilde{P}_{t+1}^{*}}{(\widetilde{P}_{t}^{*})^{2}} \left(\frac{\widetilde{P}_{t+1}^{*}}{\widetilde{P}_{t}^{*}} - 1\right) P_{t+1}Y_{t+1}$$

Optimal Price

• Putting it all together yields (exercise: check)

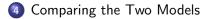
$$\left(\frac{(\widetilde{P}_{t}^{*})}{P_{t}}\right)^{-\epsilon} Y_{t} - \frac{\widetilde{P}_{t}^{*}}{P_{t}} \epsilon \left(\frac{(\widetilde{P}_{t}^{*})}{P_{t}}\right)^{-\epsilon-1} Y_{t} + TC_{t}'(Y_{t}(j))\epsilon \left(\frac{(\widetilde{P}_{t}^{*})}{P_{t}}\right)^{-\epsilon-1} Y_{t} \frac{1}{P_{t}} - \lambda \left(\frac{(\widetilde{P}_{t}^{*})}{(\widetilde{P}_{t-1}^{*})} - 1\right) \frac{Y_{t}P_{t}}{(\widetilde{P}_{t-1}^{*})} + \lambda \mathbb{E}_{t} \left[Q_{t \to t+1} \frac{(\widetilde{P}_{t+1}^{*})}{(\widetilde{P}_{t}^{*})^{2}} \left(\frac{(\widetilde{P}_{t+1}^{*})}{(\widetilde{P}_{t}^{*})} - 1\right) Y_{t+1}P_{t+1}\right] = 0$$

Roadmap











Cross-Sectional Price Dispersion

- Is there a cross-section of different prices across the varieties?
- Rotemburg: none. Why? Firms will just re-adjust each period. All the same, so they set the same price.
- Calvo: in general, there will be dispersion. E.g. consider initial price index *P*₀.

$$\Rightarrow P_{1} = \left[\theta(P_{0})^{1-\epsilon} + (1-\theta)(P_{1}^{*})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \Rightarrow P_{2} = \left[\theta\{\theta(P_{0})^{1-\epsilon} + (1-\theta)(P_{1}^{*})^{1-\epsilon}\} + (1-\theta)(P_{2}^{*})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = \left[\theta^{2}(P_{0})^{1-\epsilon} + \theta(1-\theta)(P_{1}^{*})^{1-\epsilon} + (1-\theta)(P_{2}^{*})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \Rightarrow P_{3} = \left[\theta^{3}(P_{0})^{1-\epsilon} + \theta^{2}(1-\theta)(P_{1}^{*})^{1-\epsilon} + \theta(1-\theta)(P_{2}^{*})^{1-\epsilon} + (1-\theta)(P_{3}^{*})^{1-\epsilon}\right]$$

and so on.

Induced Distortions

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• Adjustment costs in Rotemburg are goods that come out of the resource constraint

$$Y_{t} = C_{t} + \int_{0}^{1} AC_{t}(j)dj$$
$$= C_{t} + \int_{0}^{1} \frac{\lambda}{2} \left(\frac{(\widetilde{P}_{t}^{*})}{(\widetilde{P}_{t-1}^{*})} - 1\right)^{2} Y_{t}dj$$
$$= C_{t} + Y_{t}\frac{\lambda}{2} \int_{0}^{1} \left(\frac{P_{t}}{P_{t-1}} - 1\right)^{2} dj$$
$$= C_{t} + Y_{t}\frac{\lambda}{2} \left(\frac{P_{t}}{P_{t-1}} - 1\right)^{2}$$
$$\Rightarrow Y_{t} = \left\{1 - \frac{\lambda}{2} \left(\frac{P_{t}}{P_{t-1}} - 1\right)^{2}\right\}^{-1} C_{t}$$

i.e. there is an "inefficiency wedge" between output and consumption. $\frac{22/27}{22}$

Induced Distortions

- Under the Calvo model, price dispersion creates distortions.
- Recall from the production function that

$$Y_t(j) = A_t N_t(j)^{1-lpha}$$

 $\Rightarrow N_t(j) = \left(rac{Y_t(j)}{A_t}
ight)^{rac{1}{1-lpha}}$

Which is labour demand for firm j. Aggregation gives

$$N_t = \int_0^1 N_t(j) dj = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{1-\alpha}} dj$$
$$= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj$$

where the last line comes from plugging-in j's demand function.

Induced Distortions

Notice that if there is perfect price flexibility, then

$$\int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj = \int_0^1 \left(\frac{P_t}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj = 1$$

• With rigidities though, see that

$$N_t^{1-\alpha} = \left(\frac{Y_t}{A_t}\right) \left\{ \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj \right\}^{1-\alpha}$$
$$\Rightarrow Y_t = A_t N_t^{1-\alpha} \left\{ \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj \right\}^{\alpha-1}$$

where $\left\{\int_{0}^{1} \left(\frac{(P_{t}^{*})}{P_{t}}\right)^{\frac{-\epsilon}{1-\alpha}} dj\right\}^{\alpha-1} < 1$ [you need not show this].

• You can interpret the last equality as an aggregate production function.

Big Picture

- We want a model with price rigidity so we can think about non-neutral monetary policy.
- Ok, but what does price stickiness itself imply about welfare?
- It's a bad thing.
- It's a friction: firms are unable to update their prices freely, even if they wanted to.
- This can only hurt our economy relative to a benchmark without price rigidity.

Big Picture

- The setup of these two models allows us to think a bit about what the welfare cost of this friction is.
- In Calvo, the dispersion hurts welfare.
- In Rotemburg, the adjustment costs hurt welfare.

Roadmap





3 Rotemberg Model







- We looked at two forms of modelling price stickiness.
- Both create distortions.
- Different sources of distortion though.