Applied Computational Economics
Lab 1
Solving Representative Agent Partial Equilibrium Models

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Q1

Consider the following household optimisation problem

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$\begin{aligned} c_t + i_t &= rk_t + ws_t \\ i_t &= k_{t+1} - (1-\delta)k_t \\ s_t &\sim G(s_t|s_{t-1}) \\ k_{t+1} &\geq 0. \end{aligned}$$

where c_t, k_t, i_t, r, w are consumption, capital and investment, the return to capital and labour at time t respectively. Notice that r and w are taken as exogenous parameters in this problem. The variable s_t is an exogenous variable, which affects the labour earnings of the household. The function $G(s_t|s_{t-1})$ is a stochastic process describing the distribution over s_t, conditional on s_{t-1}. The preference parameters are such that $\beta \in [0,1]$ and $\sigma \in \mathbb{R}^+$. The household's labour supply is normalised to unity; they receive no disutility from it.

(a) Re-cast this problem as a Bellman equation.

(b) Solve this problem using gridsearch. Use the parameters given in table 1. Assume that the labour earnings shock is deterministic: $s_t = 1 \ \forall t$. Use a lower-bound on the capital stock of $\underline{k} = 0.0001$ and an upper-bound of $\bar{k} = 1$ in your solution. Solving using 501 gridpoints for the capital stock. Plot the solution for the k' policy and value functions.

(c) Re-solve part (b) using a value of $r = 0.25$. How do the solutions differ? Why?

(d) Now re-solve part (c) with $\bar{k} = 100$. How do the solutions differ? Why?

(e) Comment on the partial equilibrium assumption setup of the problem. How does it relate to what we think of as a steady state?

Parameter	**Value**
β	0.95
σ	2.00
r	0.01
w	1.00
δ	0.10

Table 1: Parameterisation

Q2

Stick with the model setup from the previous problem where now we allow s_t to be a stochastic process. Return to the parametrisation given in table 1 while also making the following assumptions regarding the process s_t. It follows a two-state Markov process where it can assume the values $s_t \in \{0.95, 1.05\}$ and has transition matrix of the form

$$Q_s = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$$

Solve the household's problem using gridsearch where $\bar{k} = 1$. Plot the solution for the k' policy function for both values of s_t. Compare with that from Q1.

Q3

Return to the deterministic version, (from Q1), of the model. Solve the household's problem using piecewise linear interpolation instead of gridsearch. Solve using 31 gridpoints for capital. Then try 51 gridpoints and finally 101. Comment on precision of the solution. Plot the value and policy functions. Compare with those from Q1.