Applied Computational Economics
Problem Set 2
Solving Representative Agent General Equilibrium Models

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Q1

In this problem we consider a production economy in general equilibrium. The household owns the capital stock and supplies labour, with a problem of the form

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$\begin{aligned} c_t + i_t &= R_t k_t + W_t + \Pi_t \\ i_t &= k_{t+1} - (1-\delta)k_t \\ k_{t+1} &\geq 0. \end{aligned}$$

where c_t, k_t, i_t R_t, W_t, Π_t are consumption, capital, investment, the return to capital, the return to labour and profit distributions at time t respectively. The preference parameters are such that $\beta \in [0,1]$ and $\sigma \in \mathbb{R}^+$. The household's labour supply is normalised to unity; they receive no disutility from it. The firms in the economy produce using capital and labour in a constant returns to scale production function, with a problem of the form

$$\Pi_t = \max_{\{K_t,N_t\}} K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - R_t K_t$$

where K_t and N_t refer to the aggregate capital and employment in the economy respectively and $\alpha \in [0,1]$. Let's first characterise the optimal behaviour of agents in the model and define the equilibrium.

1. Write down the household's recursive formulation.
2. Write down the firm's optimality conditions.
3. Define a recursive competitive equilibrium (RCE) for this economy.

Now let's think about computing the RCE. We will use the parameterisation given in table 1. To keep the problem tractable, (without needing to resort to parallel computing), we'll solve for the equilibrium in a relatively close neighbourhood to the model's steady state.

4. Find an analytical expression for the steady state capital stock of the economy (K^{SS}).

Parameter	Value
β	0.95
σ	2.00
δ	0.10
α	0.33

Table 1: Parameterisation

We'll restrict our attention to the global solution to the model 5% on either side of the steady state. That is — set the lower-bound for capital $\underline{K}$ to be $0.95K^{SS}$ and the upper-bound $\bar{K}$ to be $1.05K^{SS}$. Then your grid for capital will be evenly-spread across the interval $[\underline{K}, \bar{K}]$.

5. Solve for the RCE by using piecewise linear interpolation on the household's value function. Use 7 gridpoints for the capital stock and update the capital law of motion slowly. Plot the equilibrium $G(K)$.

6. Solve for the RCE by using gridsearch. Use 21 gridpoints for the capital stock. Then try 51 gridpoints. Then try 71 gridpoints Compare the convergence properties of the gridsearch approach with that of interpolation. What can you conclude?