Applied Computational Economics
Problem Set 3
Solving Heterogeneous Agent General Equilibrium Models with Idiosyncratic Uncertainty

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Q1

In this problem, we'll solve the incomplete asset markets endowment model with household heterogeneity of Huggett (1993). Households are endowed with consumption goods each period (denoted by s_t); these goods serve as the numerairé in the economy. Households can save and borrow through riskless assets denoted by a_{t+1}. A positive value of this variable denotes savings while a negative value denotes borrowing. Household borrowing is subject to an exogenous limit denoted by $\underline{a}$. The household problem can be cast as

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$\begin{aligned} c_t + a_{t+1} &= R_t a_t + s_t \\ a_{t+1} &\geq \underline{a} \\ s_t &\sim G(s_t | s_{t-1}) \end{aligned}$$

where R_t denotes the riskless rate of return on assets and $G(s_t|s_{t-1})$ is the stochastic process governing the endowment of consumption goods. Assume that there is a unit mass of these households.

1. Write down the household's Bellman equation.
2. Assume that the economy starts with some initial distribution across states of $\mu_0(\mathcal{A}, \mathcal{S})$. How does this distribution evolve over time?
3. Assume that the riskless assets are in zero net supply. What are the market clearing conditions of this economy?
4. Define a recursive competitive equilibrium (RCE) in this model.
5. Solve for the steady state RCE using gridsearch across the household's optimisation problem with 301 asset gridpoints. The lower-bound for the discretised asset space should be the $\underline{a}$ above; denote the upper-bound as $\bar{a}$. Assume that the process for the endowment is such that $s_t \in \{s^L, s^H\}$ and follows a Markov process with transition matrix denoted by G. Use the parameterisation given in table 1. Plot the savings policy functions for households in the $a - a'$ space for the two endowment levels. Plot the histograms over assets for the two endowment levels. Plot the empirical distribution functions over assets for the two endowment levels using the *cumsum* command. How do the distributions differ across the two endowment levels?

6. What is the equilibrium riskless rate in the economy?

7. Check both market clearing conditions: do they both hold numerically?

Parameter	**Value**	**Parameter**	**Value**
β	0.95	$\bar{a}$	4.00
σ	2.00	$G(s^L, s^L)$	0.90
s^L	0.50	$G(s^L, s^H)$	0.10
s^H	1.00	$G(s^H, s^L)$	0.10
$\underline{a}$	-1.00	$G(s^H, s^H)$	0.90

Table 1: Parameterisation