# Applied Computational Economics
# Problem Set 3
# Solving Heterogeneous Agent General Equilibrium Models with Idiosyncratic Uncertainty

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## Q1

1. The Bellman equation is across their asset holdings and their endowment.

$$v(a, s) = \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}[v(a', s')]$$
$$c = Ra + s - a'$$
$$a' \geq \underline{a}.$$

   We'll denote their optimal control for $a' = g(a, s)$ and $c = c(a, s)$ in what follows.

2. There will be a cross-sectional distribution, (since there is a unit mass of households), across the state space of the model $\mathcal{A} \times \mathcal{S}$. It evolves over time according to law of motion

$$\mu_{t+1}(a', s') = \int_{\mathcal{S}\times\mathcal{A}} \mathbb{1}_{a'=g(a,s)} G(s'|s)\mu_t(da, ds) \tag{1}$$

   for all $a' \in \mathcal{A}$ and $s' \in \mathcal{S}$.

3. There are two market clearing conditions — one for the asset market and another for the consumption goods market. They're such that

$$\int_{\mathcal{S},\mathcal{A}} [s - c(a, s)]\mu(da, ds) = 0 \tag{2}$$
$$\int_{\mathcal{S},\mathcal{A}} [g(a, s)]\mu(da, ds) = 0, \tag{3}$$

   which say that the aggregate endowment of consumption goods equals aggregate consumption and that aggregate asset holdings are in zero net supply respectively. This zero net supply condition has the interpretation that, in the aggregate, savings equal borrowings. We can think of these riskless bonds as households lending to one another.

4. A (steady state) recursive competitive equilibrium here is defined as a set of objects $v(a, s)$, $c(a, s)$, $g(a, s)$, $\mu(a, s)$ and $R$ such that

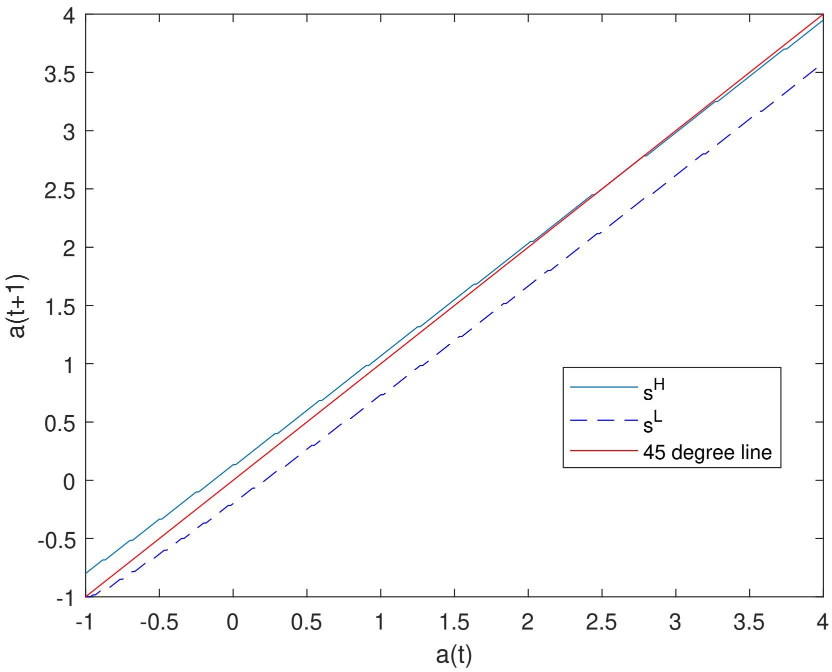


Figure 1: Savings policy functions

- $v(a, s)$ is the household's value function,
- $g(a, s)$ and $c(a, s)$ are the household's optimal controls for $a'$ and $c$ respectively,
- $R$ is the steady state riskless rate,
- $\mu(a, s)$ is the stationary distribution of agents across the state space $\mathcal{A} \times \mathcal{S}$, which evolves according to equation (1),
- The consumption goods and riskless bonds markets clear as per equations (2) and (3).

5. The policy functions are presented in figure 1. Just one thing I'll quickly point-out about the code to re-iterate from the lecture. Since the control for $a'$ is subject to this exogenous lower bound $\underline{a}$, to account for the constraint, all we need to do is make the $\underline{a}$ the lowest possible gridpoint for assets. The histogram and empirical distribution functions (EDF) are plotted in figure 2. Notice that to generate the EDF, you'd find the cumulative sum for any given gridpoint and then divide by the overall sum for the endowment level under consideration. I created these figures using the following code

```
plot(a_grid,cumsum(mu(:,1))/sum(mu(:,1)))
plot(a_grid,cumsum(mu(:,1))/sum(mu(:,1)))
hold on
plot(a_grid,cumsum(mu(:,2))/sum(mu(:,2)),'r')
xlabel('a(t)')
```

```
ylabel('Distribution')
legend('s^{H}','s^{L}')
```

where $mu(:,1)$ denotes the $s^H$ distribution over assets and $mu(:,2)$ is that for $s^L$.

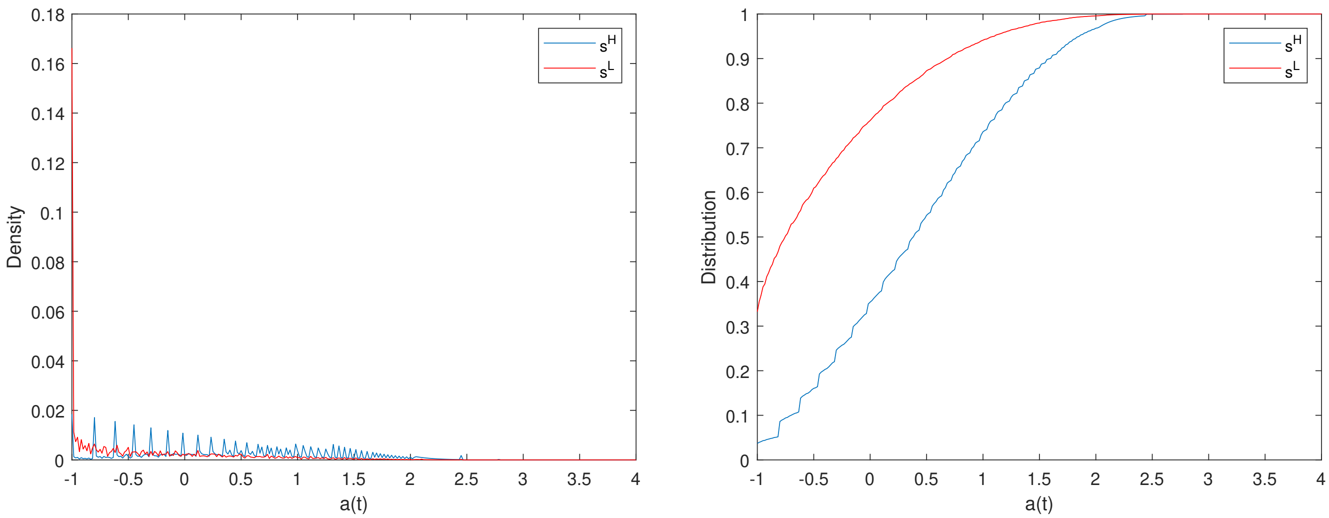


Figure 2: Histograms and empirical distribution functions

As intuition would suggest, households that have the low endowment level tend to borrow more than those with the high endowment. You can see this from figure 2 in so far as there is a large mass of households that are pinned to their exogenous debt limit $\underline{a}$.

6. I find the net riskless rate to be 2.05%.

7. Yes both markets clear numerically. I find aggregate consumption and endowment to both be 0.75, while the aggregate assets are zero. Note that the asset market clears by design in the algorithm (by iterating and updating on the riskless rate), while the goods market clears by Walras' law, (i.e. we didn't make it clear explicitly in the code).