Applied Computational Economics
Problem Set 4
Solving Heterogeneous Agent General Equilibrium Models with Aggregate Uncertainty

The University of Nottingham

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Q1

Here we'll try to go for a rough solution to the Krusell and Smith (1998) model with aggregate uncertainty. The recursive formulation for the household is the following

$$v(k, e; z, \mu) = \max_{c,k'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{e',z'}[v(k', e'; z', \mu')]$$

subject to

$$\begin{aligned} c + k' &= w(z, K)\bar{e}e + \{1 - \delta + r(z, K)\}k \\ \mu' &= G(\mu, z, z') \\ k &\geq 0 \end{aligned}$$

where the first constraint is that for the household's budget, the second is the law of motion the cross-sectional distribution and the third ensures capital's non-negativity. The variable e_t is an idiosyncratic employment shock, such that $e_t \in \{0, 1\}$, which denotes unemployed and employed respectively. The parameter $\bar{e} \in [0, 1]$ is an exogenous work time endowment for the household. That is — since the labour supply doesn't enter the utility function of the household directly, when employed the household will supply $\bar{e}$ in the labour market. Object μ is the cross-sectional distribution of agents across the space of capital and employment shocks. Variables $w(z, K)$ and $r(z, K)$ are the wage and rental rate for capital respectively. K denotes the aggregate capital stock and z denotes aggregate productivity. Assume that the firms produce using the production function

$$Y = zK^{\alpha}L^{1-\alpha}$$

where Y denotes aggregate output and L denotes aggregate labour.

1. Re-write the above recursive formulation in the language of Krusell and Smith (1998). Explain any concepts relating to approximations and the like.
2. Write-down the factor market clearing conditions for this economy.
3. Explain the steps involved in the Krusell and Smith (1998) computational algorithm.

4. Assume that $e_t \in \{0, 1\}$ and that $z_t \in \{1.01, 0.99\}$. These two stochastic variables are assumed to evolve with a joint Markov process over time. We'll use the Markov process given in the file on the course website called *PS4_transition_matrix.m*. This matrix outputs the joint transition, called Q_trans. This matrix is structured such that rows and columns read pairs (z(1), e(1)), (z(1), e(2)), (z(2), e(1)), (z(2), e(2)) where $z(1) = 1.01$, $z(2) = 0.99$, $e(1) = 0$, $e(2) = 1$. The way you can visualise this matrix is as follows

$$Q(z', e'|z, e) = \begin{bmatrix} q(z(1), e(1)|z(1), e(1)) & q(z(1), e(2)|z(1), e(1)) & q(z(2), e(1)|z(1), e(1)) & q(z(2), e(2)|z(1), e(1)) \\ q(z(1), e(1)|z(1), e(2)) & q(z(1), e(2)|z(1), e(2)) & q(z(2), e(1)|z(1), e(2)) & q(z(2), e(2)|z(1), e(2)) \\ q(z(1), e(1)|z(2), e(1)) & q(z(1), e(2)|z(2), e(1)) & q(z(2), e(1)|z(2), e(1)) & q(z(2), e(2)|z(2), e(1)) \\ q(z(1), e(1)|z(2), e(2)) & q(z(1), e(2)|z(2), e(2)) & q(z(2), e(1)|z(2), e(2)) & q(z(2), e(2)|z(2), e(2)) \end{bmatrix}$$

where each little q is a conditional probability. The sum across rows comes to unity. We'll take one period in the model to be a month. This file also contains the transition matrix for the productivity state only $Q(z'|z)$, which it calls Q_agg. Using the parametrisation in table 1, implement the Krusell and Smith (1998) algorithm. Use an upper bound for k of $\bar{e}\frac{1-\alpha}{\alpha} * 2$.

Parameter	**Value**
β	0.99
σ	2.00
δ	0.02
α	0.36
$\bar{e}$	0.33

Table 1: Parameterisation