# Applied Computational Economics
# Problem Set 4
# Solving Heterogeneous Agent General Equilibrium Models with Aggregate Uncertainty

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## Q1

1. We re-write the problem by using the average capital stock as a proxy for the cross-sectional distribution. Specifically that

$$v(k, e; z, K) = \max_{c,k'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{e',z'}[v(k', e'; z', K')]$$

subject to

$$\begin{aligned} c + k' &= w(z, K)\bar{e}e + \{1 - \delta + r(z, K)\}k \\ \log((K')^1) &= b^0_{z_i} + b^1_{z_i} \log(K^1) \\ k &\geq 0 \end{aligned}$$

where $K^1$ denotes the mean of the capital distribution and $z_i$ denotes the two possible productivity levels $z_1 = 1.01$ and $z_2 = 0.99$.

2. Just watch out for this $\bar{e}$ thing. It's annoying, but standard to have this parameter in these models.

$$\begin{aligned} L &= \bar{e} \int_{e,k} e d\mu \\ K &= \int_{e,k} k d\mu. \end{aligned}$$

The thing to notice for the labour clearing condition is that there will always be a constant fraction of households that are employed and unemployed given the current state of technology $z$. That constant fraction can be found from taking the joint transition matrix $Q(e', z'|e, z)$ and raising it to the power of some large number. Say 1000. This gives you the stationary fractions of employment levels across each state. We can then write that labour in a given state of $z$ is

$$L = \bar{e}(1 - u^i)$$

where $u^i$ is the fraction of unemployed households when $z = z_i$.

3. The algorithm is as follows. Firstly, decide on the number of households to simulate $i \in \{1, 2, ..., N\}$ and $t \in \{1, 2, ..., T\}$. Then do the following.
   (a) Conjecture the initial regression coefficients $\vec{b}_{z_i} = (b^0_{z_1}, b^1_{z_1}, b^0_{z_2}, b^1_{z_2})$.
   (b) Given this conjecture, solve the household's Bellman equation. Note that the households take the wages and capital returns as given: they come from the level of $K$.
   (c) Simulate random draws just for the aggregate state using Q_agg for the $T$ periods.
   (d) Given the values of the aggregate state, simulate draws from the individual state for each of the $N$ households. Notice that this effectively splits the entire transition matrix into four smaller 2x2 matrices for the individual state, (top left, top right, bottom left, bottom right). Don't forget to re-normalise the probabilities. The households all need a starting capital stock. I just set it equal to half the upper bound for $K$.
   (e) Use the policy functions from step (b) and the stochastic variables drawn to simulate your panel dataset.
   (f) Regress the average capital stock for a given $t$ against its previous value for $t-1$. Do this separately for the $z(1)$ and $z(2)$ draws.
   (g) Update the regression coefficients and return to step (b). Repeat until convergence.

4. I use 21 capital gridpoints and $N = 2000$ with $T = 6000$ and a burn-in of 1000 periods, (tossed observations to remove the effect of starting points). This gives $R^2$ of about 0.91 for the two sets of regression coefficients with $(b^0_{z_1}, b^1_{z_1}) = (0.0063, 0.9499)$ and $(b^0_{z_2}, b^1_{z_2}) = (0.0056, 0.9546)$. Note the low $R^2$ comes from the crappy approximation. If you interpolate you can probably do a lot better.